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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

RAINFALL STUDIES FOR NEW YORK, N. Y.

By S. D. Bleich,[1] Esq.

Synopsis

The object of this paper is to collect rainfall data, arrange them in order of intensities, and then derive formulas and graphs which may be used for purposes of sewer design and investigations. Statistics of the heavier rainfalls have been culled from the records of the Central Park gauge in New York, N. Y., covering the period from 1869 to 1930, inclusive. The intensities for various durations are then listed in order of their magnitudes and several types of precipitation formulas are investigated. By comparison of the results the modified exponential type proves most nearly to accord with the observed data. With suggested constants this is recommended for use in New York City and, when properly adjusted as to values of coefficients, to other localities as well.

For the proper design of storm sewers it is essential to know fairly definitely the frequency at which the severe rainfall intensities occur for different intervals of time. The rate of rainfall is one of the factors used in estimating run-off in the determination of sewer capacities and the choice of its value will be governed by the frequency. The most severe intensities of rain are not generally used in obtaining the capacities of sewers, as the cost of such sewer sizes would be excessive and, in fact, unwarranted because of the rarity of these storms.

Severe rainfalls are listed in terms of the frequency of their occurrence. Further, it is necessary to classify them by their duration, by their rate or intensity corresponding to the duration, and by the frequency that such rates or intensities have been equalled or exceeded.

Rainfalls are designated by referring to the frequencies at which certain intensities occur for various periods. For instance, a "2-year" storm is one in which the relation between rate of rainfall and corresponding duration has been equalled or exceeded on an average of once in two years for the entire recorded period.

Note.—Discussion on this paper will be closed in May, 1934, *Proceedings*.

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Rainfall Data and Their Uses

It is apparent that the first task is to collect the basic data. This, in many respects, is the most difficult part of the work as the records for any one rain-gauge are enormous. Those from the Government Observatory in Central Park, New York City, which extend over 62 years, from 1869 to 1930, inclusive, and form the subject of this paper, required a great deal of patience and painstaking labor in tabulating the data as correctly as possible.[2] The earlier records do not give as complete details for short-time intensities as the more recent ones, and this must be borne in mind in interpreting them.

A summary in Table 1 shows the limitations found or assumed in collecting the data, while the storm records themselves are listed in Table 2. It is evident that there is no regularity in the recurrence of severe intensities for any period. In order, therefore, to obtain working rules, it is desirable to arrange all intensities in a regular sequence starting with the heaviest and proceeding to the lowest. This has been done in Table 3.

TABLE 1.—Scope of Rainfall Tabulation for Central Park Gauge. New York City, 1869 to 1930, Inclusive

Period, in minutes	Maximum observed intensity, in inches per hour	Minimum intensity considered, in inches per hour *
5	9.12	3.00
10	9.06	1.92
15	8.44	1.92
30	5.50	0.91
60	3.31	0.90
120	1.85	0.90

* Except years in which the maximum is less than the minimum given.

Since the record covers 62 years, the frequency (second column), corresponding to the first number, is $\frac{1}{62}$, or 0.016 times per year. A "5-year" storm lies between Serial Nos. 12 and 13, as one-fifth of 62 is 12.4.

Summarized from Table 3, the character of the various kinds of storms are as given in Table 4. Where the serial number is a fraction the rainfall intensities have been interpolated.

As the data in Table 3 are limited, and known as "random samples," any one sample may not conform to the general trend of the aggregate samples. This is readily indicated by the irregular lines when the data are plotted on cross-section paper.

The human mind strives to secure order in the relations of physical measurements and considers that the apparent irregularity is due to errors in the measurements, to the incompleteness of the observations, or to both. To overcome such apparent deficiencies resort is made to various methods. A smooth curve may be drawn which would average the plotted data, or an equation may be developed by methods used in statistics or biometrics. In this study the graphical and algebraic methods have been used.

[2] Data collected by Mr. Hufeland, *Engineering News*, August 31 and September 7, 1916. covering the period, 1869 to 1913, inclusive, supplemented by published records of the U. S. Weather Bureau. Gaps in these data indicate that such maximum rainfall intensities were not available.

TABLE 2.—BASIC CHRONOLOGICAL RECORDS OF HEAVY RAINFALLS, CENTRAL PARK GAUGE, NEW YORK CITY, 1869 TO 1930. INTENSITY OF RAINFALL, IN INCHES PER HOUR, FOR DURATIONS GIVEN

Year	Date	Duration, in Minutes: 5	10	15	30	60	120
1869	6/21	....	2.10	....	1.20	....	....
	8/13	5.88	3.12	2.24	....	....	....
	8/15	5.88	2.22	....	....	....	....
1870	6/27	....	2.22	2.00	1.52	....	....
	7/28	4.32	2.70	....	....	....	....
1871	6/ 7	4.20	3.54	3.12	1.82	....	....
	7/16	3.00	3.00	2.88	2.04	1.30	....
	8/16	....	....	2.00	....	....	....
	9/26	....	2.28	2.28	1.62	0.97	....
	10/11	6.48	4.38	3.60	2.20	1.14	....
	10/26	3.24	2.94	2.88	1.82	....	....
1872	7/25	....	....	....	1.14	1.00	....
1873	7/ 5	3.96	3.00	2.48	1.54	....	....
	7/27	3.24	2.28	1.92	1.40	....	....
	8/21	....	2.46	....	1.30	1.00	....
1874	7/10	3.00	2.22	....	1.14	0.98	....
	7/12	4.20	3.00	2.40	1.34	....	....
1875	6/22	3.48	2.10	....	....	....	....
	7/26	5.00	5.00	4.56	2.54	1.27	....
	8/18	3.72	3.30	2.64	1.84	1.25	....
1876	5/26	....	2.70	....	....	....	....
	8/16	3.36	3.24	3.00	2.28	1.09	....
1877	6/21	....	....	....	1.00	....	....
	7/ 3	3.36	2.58	2.00	....	....	....
	7/17	3.20	3.18	3.12	2.00	1.09	....
	8/14	....	....	2.00	1.40	....	....
	8/25	....	2.58	....	....	....	....
	10/ 4	....	2.00	2.00	1.80	1.50	1.08
1878	6/21	....	....	....	1.36	1.00	....
	7/10	....	2.16	....	....	....	....
	7/21	....	2.10	....	....	....	....
	7/31	....	....	....	1.48	....	....
	8/ 1	....	2.22	2.16	1.72	1.12	....
	8/ 5	9.00	8.22	8.00	5.22	2.66	....
	9/ 4	3.60	2.88	2.08	1.72	1.50	1.10
	11/27	....	....	....	1.00	....	....
1879	8/25	3.00	2.40	2.00	1.08	....	....
1880	7/12	6.00	4.74	3.44	2.04	1.17	....
	7/27	4.32	3.00	2.04	....	....	....
	8/10	3.00	....	....	....	....	....
	8/20	5.40	2.88	1.96	....	....	....
1881	5/22	5.04	4.68	4.68	2.40	1.20	....
	7/ 4	4.20	2.58	1.92	....	....	....

Year	Date	Duration, in Minutes: 5	10	15	30	60	120
1881	8/ 7	3.36	2.28	....	....	....	....
1882	6/15	4.00	....	....	....	....	....
	6/26	....	....	1.92	....	....	....
	9/11	....	2.10	1.92	1.62	1.10	....
	9/20	3.96	2.76	2.08	....	....	....
	9/22	3.00	2.82	2.68	2.22	1.34	....
	9/23	3.00	3.00	2.64	2.30	1.74	1.31
1883	5/21	3.00	2.52	2.32	1.86	1.51	....
	6/ 6	4.44	....	....	....	....	....
	7/13	5.16	3.48	2.84	1.56	....	....
1884	6/25	....	2.04	1.92	1.00	1.00	....
	7/ 4	3.00	....	....	....	....	....
	7/12	....	2.28	....	1.10	....	....
	8/ 5	4.20	3.18	2.48	1.36	....	....
	8/22	3.72	3.54	3.24	2.70	1.51	....
	8/30	3.00	2.88	2.72	1.90	1.01	....
1885	6/ 5	....	2.16	....	....	....	....
	7/29	3.48	2.10	....	....	....	....
	8/ 3	3.60	3.12	2.56	1.82	1.00	....
1886	11/18	3.36	....	....	....	....	....
1887	5/31	3.36	3.18	2.52	2.22	1.22	....
	6/17	3.36	....	....	....	....	....
	6/20	4.20	2.58	1.84	1.08	....	....
	6/22	3.12	2.52	2.04	1.40	1.08	....
	8/18	4.92	2.58	....	....	....	....
	8/21	3.60	2.10	....	....	....	....
	8/22	....	....	....	1.00	0.92	....
	8/24	3.00	2.16	....	....	....	....
1888	5/28	....	....	....	1.20	....	....
	5/29	....	....	1.60	....	....	....
	6/26	3.30	2.82	2.48	1.78	....	....
	7/19	3.00	2.46	2.08	1.12	....	....
	8/ 4	....	2.52	2.40	2.20	1.45	1.02
	8/21	2.76	2.40	2.28	1.62	1.21	....
	9/ 8	....	....	....	1.54	....	....
	9/20	3.00	3.00	....	1.76	1.21	....
1889	6/11	....	1.98	....	....	....	....
	6/15	....	....	....	1.10	....	....
	7/27	....	....	....	1.20	....	....
	7/30	3.60	2.40	2.20	1.36	1.00	....
	7/31	3.00	2.10	....	....	....	....
	8/ 2	3.00	....	....	....	....	....
	9/17	3.24	2.70	2.24	....	....	....

Year	Date	Duration, in Minutes: 5	10	15	30	60	120
1890	6/ 6	3.60	3.48	2.48	1.50	....	....
	7/17	....	2.28	....	....	....	....
	8/22	3.48	2.40	1.96	1.20	....	....
	9/16	2.88	2.04	....	....	....	....
	9/17	....	....	....	1.00	....	....
1891	4/23	3.12	....	....	....	....	....
	6/17	2.88	2.28	....	1.00	....	....
	6/23	3.00	2.64	2.32	1.20	....	....
	8/23	....	2.28	....	1.22	1.04	....
	9/ 5	....	2.00	....	1.10	....	....
1892	6/ 9	3.00	....	....	....	....	....
	7/ 3	7.44	4.44	3.04	1.54	1.13	....
	8/ 9	....	....	....	1.06	....	....
	9/13	3.72	2.70	2.18	1.12	....	....
1893	6/ 6	3.84	3.60	2.44	1.28	1.08	....
	7/26	3.84	2.40	....	....	....	....
	8/19	5.04	3.72	2.76	1.74	1.07	....
	8/20	6.00	5.10	4.80	3.30	1.68	0.92
	8/23	....	....	....	1.20	1.05	....
	9/ 7	....	2.10	....	....	....	....
	10/27	3.00	1.98	....	1.08	....	....
1894	7/29	....	2.40	....	....	....	....
	8/ 3	4.80	2.70	....	1.00	....	....
	9/ 8	3.60	2.70	2.64	1.78	1.03	....
	9/16	....	2.28	....	....	....	....
1895	6/ 5	5.64	3.72	2.60	1.38	....	....
	8/ 4	....	....	....	1.16	....	....
	8/ 6	6.35	3.96	3.12	....	....	....
	8/31	3.96	3.42	2.80	1.64	....	....
1896	5/28	....	....	....	1.04	....	....
	6/10	4.32	2.64	2.00	....	....	....
	7/ 6	4.20	2.70	2.64	2.12	1.85	....
	7/15	3.60	2.00	....	1.04	....	....
	7/20	3.00	2.10	....	1.22	....	....
	7/22	3.24	2.04	2.00	1.20	0.92	....
	7/30	....	2.40	....	....	....	....
	9/13	3.90	3.90	2.92	1.70	0.94	....
	9/15	3.60	2.04	....	....	....	....
	9/18	4.68	2.64	....	....	....	....
1897	5/24	5.52	3.72	3.24	2.20	1.18	....
	6/ 9	2.88	2.10	....	1.28	1.02	....
	7/21	3.00	....	....	....	....	....
	7/23	....	2.34	2.34	1.40	1.06	....

TABLE 2.—(*Continued*)

Year	Date	Duration, in Minutes: 5	10	15	30	60	120
1897	7/28	....	....	....	1.00	....	....
	8/23	3.00	3.00	2.76	2.22	1.74	0.96
	9/ 1	4.20	3.00	2.32	1.44	....	....
1898	4/24	3.60	2.28	....	....	....	....
	7/ 4	4.20	3.00	2.00	....	....	....
1899	6/15	....	....	....	1.62	....	....
	6/28	3.00	....	....	....	....	....
	7/12	4.20	3.18	2.16	....	....	....
	8/10	5.28	2.76	1.96	1.18	....	....
	9/ 1	....	....	....	1.42	1.20	0.90
	9/ 3	4.80	3.00	2.80	1.70	....	....
1900	5/19	....	2.00	1.92	1.58	0.95	....
	8/12	3.00	....	....	....	....	....
	8/26	3.84	3.30	3.20	2.30	....	....
1901	7/ 5	3.48	2.88	2.72	2.00	1.50	1.32
	7/ 6	4.32	3.66	3.00	1.54	....	....
	7/31	6.98	6.00	4.20	2.16	1.10	....
	8/24	....	....	....	1.54	1.08	....
1902	6/21	....	....	....	1.00	....	....
	7/20	....	2.10	....	....	....	....
	7/28	6.00	5.16	4.60	4.08	3.02	....
	8/ 5	6.00	5.16	4.52	2.90	1.61	....
	8/19	....	2.50	2.40	....	....	....
1903	2/28	4.08	....	....	....	....	....
	6/14	....	2.00	2.00	1.42	....	....
	6/21	3.12	2.28	....	....	....	....
	6/29	2.88	2.70	2.60	1.82	1.55	1.05
	7/ 2	....	2.40	....	....	....	....
	7/22	3.60	2.88	2.00	1.04	....	....
	9/16	....	....	....	1.06	1.03	....
	9/17	....	1.98	....	....	....	....
	10/ 8	3.00	2.40	2.40	2.00	1.44	....
	10/ 9	....	....	....	1.50	1.35	1.21
1904	5/30	....	2.10	....	....	....	....
	7/11	....	....	....	1.00	....	....
	7/28	3.24	....	....	....	....	....
	8/ 4	3.00	....	....	....	....	....
	8/19	3.72	3.54	3.28	2.76	1.72	1.07
	9/14	....	2.40	1.92	1.26	....	....
1905	6/22	....	2.46	2.20	1.96	1.14	....
	8/13	....	2.70	2.44	1.40	1.11	....
	10/19	....	2.16	2.08	....	....	....
1906	5/ 5	3.72	3.12	2.20	....	....	....
	5/27	5.04	3.42	2.72	....	....	....
	7/10	....	2.04	....	1.40	....	....
	7/17	3.24	3.12	2.44	....	....	....
	9/13	3.00	....	....	....	....	....
1907	8/19	3.00	....	....	....	....	....
	8/24	....	....	....	1.30	1.15	....
	9/11	3.00	3.00	2.56	1.52	....	....
1908	7/14	4.80	3.30	2.60	2.20	1.16	....
	9/28	....	....	....	1.10	....	....
1909	6/28	....	2.40	2.20	1.30	....	....
	8/ 4	....	....	....	1.32	0.96	....
	8/13	3.00	2.20	2.10	1.20	....	....
1911	3/27	3.24	1.98	....	....	....	....
	6/10	....	2.52	2.40	....	....	....
	6/11	3.00	2.60	2.40	1.54	1.28	....
	7/20	3.84	3.60	2.60	1.34	....	....
	8/15	....	2.40	2.08	1.56	....	....
	8/24	....	....	....	1.36	1.08	....
	10/19	....	2.00	2.00	1.34	1.00	....
1912	2/26	3.60	3.00	2.28	1.34	....	....
	5/30	6.60	5.28	4.12	2.16	....	....
	6/17	3.36	2.28	....	....	....	....
	7/21	3.24	2.48	2.16	1.78	0.96	....
	8/10	3.00	2.04	....	1.24	....	....
	10/12	....	2.46	2.40	1.62	1.53	0.96
1913	6/20	4.32	2.52	....	1.60	0.99	....
	7/ 1	....	2.10	1.95	....	....	....
	7/24	3.24	2.58	2.28	1.09	1.09	....
	7/28	6.30	5.94	4.88	2.98	2.74	1.56
	8/ 7	....	2.28	2.04	1.46	....	....
	9/ 4	7.20	6.90	6.40	5.24	3.31	1.85
	10/ 1	6.72	4.68	4.04	3.12	2.03	1.67
1914	7/ 2	....	....	....	1.08	....	....
	8/21	3.12	2.52	2.44	1.84	0.98	....
1915	6/15	3.24	2.64	2.00	1.24	....	....
	6/30	4.08	3.60	3.12	2.34	1.45	....
	7/ 5	6.24	5.22	3.84	2.12	1.11	....
	8/ 4	....	2.04	1.96	1.68	1.24	1.05
1916	6/17	3.36	3.06	2.80	1.48	1.09	....
	7/26	3.60	2.64	2.08	1.48	0.94	....
1917	6/14	3.24	2.70	2.32	1.88	1.12	1.10
	9/ 1	....	....	....	0.90	....	....
	12/18	....	....	....	0.96	0.95	....
1918	5/21	3.00	2.64	2.44	1.74	1.00	....
	6/ 7	3.60	2.76	2.04	1.44	1.33	....
	7/30	....	....	....	1.10	....	....
1919	6/20	6.96	5.28	4.04	2.28	1.25	....
	6/27	....	....	....	1.16	....	....
	7/26	4.80	3.60	3.16	2.24	1.33	....
	8/ 1	....	....	....	1.08	....	....
	8/ 8	....	....	....	1.16	....	....
1919	9/23	....	....	....	1.70	1.39	....
1920	7/ 3	3.96	2.46	....	1.16	1.09	....
	7/24	4.56	3.60	3.40	2.00	1.17	....
	8/11	3.60	2.64	....	1.06	....	....
1921	8/ 7	....	2.10	....	....	....	....
	9/30	5.04	3.30	2.48	1.68	1.16	....
1922	7/ 1	3.00	2.88	2.40	1.92	1.14	....
	7/ 3	4.56	3.72	3.32	1.96	1.09	....
	7/18	3.36	2.76	2.20	2.00	1.11	....
1923	6/ 7	3.00	2.16	....	....	....	....
1924	5/24	....	1.98	....	....	....	....
	8/26	....	....	....	1.12	1.02	....
1925	6/16	4.32	3.18	2.48	1.52	....	....
	6/25	4.80	3.90	3.36	1.72	....	....
	7/31	....	....	....	1.00	....	....
	8/ 9	3.36	2.70	1.92	0.98	....	....
1926	7/25	6.36	5.22	4.88	2.80	1.52	....
	8/12	4.56	4.20	3.16	2.14	2.14	1.22
	8/14	....	....	....	1.02	....	....
	9/ 6	....	1.92	....	1.24	....	....
1927	5/23	5.64	3.42	2.44	1.28	....	....
	6/26	....	....	....	1.32	....	....
	7/ 7	3.60	2.40	2.08	1.38	0.91	....
	7/23	3.84	2.70	2.20	2.00	1.15	....
	8/ 1	6.84	4.80	3.92	3.24	1.79	0.96
	8/ 8	3.00	2.76	2.76	1.68	1.20	....
	8/14	....	....	....	1.10	....	....
	10/13	3.60	3.30	2.60	1.86	1.19	....
1928	6/ 9	3.96	3.54	2.48	1.38	....	....
	6/19	....	....	....	1.20	0.95	....
	6/24	....	....	....	1.06	....	....
	7/ 6	....	....	....	1.26	....	....
	7/12	....	2.40	....	1.22	....	....
	7/20	4.14	4.14	2.96	....	....	....
	8/10	3.84	2.88	2.76	1.42	....	....
	10/19	7.56	3.78	2.52	1.36	....	....
1929	2/26	....	....	....	1.06	....	....
	3/23	3.12	2.76	2.76	1.68	1.30	....
	5/12	3.96	2.04	....	0.98	....	....
	5/24	3.00	2.70	2.32	1.22	....	....
	6/28	....	....	....	1.10	....	....
1930	6/26	....	1.92	....	0.94	....	....
	7/ 3	9.12	9.06	8.44	5.50	2.77	1.39
	7/24	3.96	3.42	2.48	1.40	....	....
	7/29	5.16	4.20	3.32	2.80	1.40	....
	8/16	5.28	3.00	....	....	....	....
	8/23	....	....	....	1.32	1.09	....

TABLE 3.—RAINFALLS IN ORDER OF MAGNITUDE, CENTRAL PARK GAUGE, NEW YORK CITY, 1869 TO 1930. INTENSITY OF RAINFALL, IN INCHES PER HOUR, FOR DURATIONS GIVEN

Serial No.	Average number per year	Duration, in Minutes					
		5	10	15	30	60	120
1	0.016	9.12	9.06	8.44	5.50	3.31	1.85
2	0.032	9.00	8.22	8.00	5.24	3.02	1.67
3	0.048	7.56	6.90	6.40	5.22	2.77	1.56
4	0.064	7.44	6.00	4.88	4.09	2.74	1.39
5	0.080	7.20	5.94	4.88	3.30	2.66	1.32
6	0.097	6.98	5.28	4.80	3.24	2.14	1.31
7	0.113	6.96	5.28	4.68	3.12	2.03	1.22
8	0.129	6.84	5.22	4.60	2.98	1.85	1.21
9	0.145	6.72	5.22	4.56	2.90	1.79	1.10
10	0.160	6.60	5.16	4.52	2.80	1.74	1.10
11	0.176	6.48	5.16	4.20	2.80	1.74	1.08
12	0.193	6.36	5.10	4.12	2.76	1.72	1.07
13	0.210	6.35	5.00	4.04	2.70	1.68	1.05
14	0.226	6.30	4.80	4.04	2.54	1.61	1.05
15	0.242	6.24	4.74	3.92	2.40	1.55	1.02
16	0.258	6.00	4.68	3.84	2.34	1.53	1.02
17	0.274	5.00	4.68	3.60	2.30	1.52	0.96
18	0.290	6.00	4.44	3.44	2.30	1.51	0.96
19	0.306	6.00	4.38	3.40	2.28	1.51	0.96
20	0.322	5.88	4.20	3.36	2.28	1.50	0.92
21	0.338	5.88	4.20	3.32	2.24	1.50	0.90
22	0.355	5.64	4.14	3.32	2.22	1.50	....
23	0.371	5.64	3.96	3.28	2.22	1.45	....
24	0.387	5.52	3.90	3.24	2.22	1.45	....
25	0.404	5.40	3.90	3.24	2.20	1.44	....
26	0.419	5.28	3.78	3.20	2.20	1.40	....
27	0.436	5.28	3.72	3.16	2.20	1.39	....
28	0.452	5.16	3.72	3.16	2.20	1.35	....
29	0.468	5.16	3.72	3.12	2.16	1.34	....
30	0.484	5.04	3.72	3.12	2.16	1.33	....
31	0.500	5.04	3.66	3.12	2.14	1.33	....

Serial No.	Average number per year	Duration, in Minutes				
		5	10	15	30	60
32	0.516	5.04	3.60	3.12	2.12	1.30
33	0.532	5.04	3.60	3.04	2.12	1.30
34	0.548	5.00	3.60	3.00	2.04	1.28
35	0.565	4.92	3.60	3.00	2.04	1.27
36	0.581	4.80	3.60	2.96	2.00	1.25
37	0.597	4.80	3.54	2.92	2.00	1.25
38	0.613	4.80	3.54	2.88	2.00	1.24
39	0.628	4.80	3.54	2.88	2.00	1.22
40	0.645	4.80	3.54	2.84	2.00	1.21
41	0.661	4.68	3.48	2.80	2.00	1.21
42	0.677	4.56	3.48	2.80	1.96	1.20
43	0.694	4.56	3.42	2.80	1.96	1.20
44	0.709	4.56	3.42	2.76	1.92	1.20
45	0.726	4.44	3.42	2.76	1.90	1.19
46	0.743	4.32	3.42	2.76	1.88	1.18
47	0.758	4.32	3.30	2.76	1.86	1.17
48	0.774	4.32	3.30	2.76	1.86	1.17
49	0.791	4.32	3.30	2.72	1.84	1.16
50	0.806	4.32	3.30	2.72	1.84	1.16
51	0.823	4.32	3.30	2.72	1.82	1.15
52	0.839	4.20	3.24	2.68	1.82	1.15
53	0.856	4.20	3.18	2.64	1.82	1.14
54	0.871	4.20	3.18	2.64	1.82	1.14
55	0.886	4.20	3.18	2.64	1.80	1.14
56	0.902	4.20	3.18	2.64	1.78	1.13
57	0.918	4.20	3.18	2.60	1.78	1.12
58	0.936	4.20	3.12	2.60	1.78	1.12
59	0.953	4.20	3.12	2.60	1.76	1.11
60	0.968	4.20	3.12	2.60	1.74	1.11
61	0.984	4.14	3.12	2.60	1.74	1.11
62	1.00	4.08	3.06	2.56	1.72	1.10

TABLE 3.—(*Continued*)

Serial No.	Average number per year	Duration, in Minutes					Serial No.	Average number per year	Duration, in Minutes				
		5	10	15	30	60			5	10	15	30	60
63	1.02	4.08	3.00	2.56	1.72	1.10	94	1.52	3.60	2.70	2.28	1.48	0.98
64	1.03	4.00	3.00	2.52	1.72	1.09	95	1.53	3.60	2.70	2.28	1.48	0.97
65	1.05	3.96	3.00	2.52	1.70	1.09	96	1.55	3.60	2.70	2.28	1.46	0.96
66	1.07	3.96	3.00	2.48	1.70	1.09	97	1.56	3.60	2.70	2.28	1.44	0.96
67	1.08	3.96	3.00	2.48	1.70	1.09	98	1.58	3.60	2.70	2.24	1.44	0.95
68	1.10	3.96	3.00	2.48	1.68	1.09	99	1.60	3.48	2.70	2.24	1.42	0.95
69	1.11	3.96	3.00	2.48	1.68	1.09	100	1.62	3.48	2.70	2.20	1.42	0.95
70	1.13	3.96	3.00	2.48	1.68	1.09	101	1.63	3.48	2.70	2.20	1.42	0.94
71	1.15	3.96	3.00	2.48	1.68	1.08	102	1.65	3.48	2.70	2.20	1.40	0.94
72	1.16	3.90	3.00	2.48	1.64	1.08	103	1.67	3.36	2.70	2.20	1.40	0.92
73	1.18	3.84	3.00	2.48	1.62	1.08	104	1.69	3.36	2.70	2.20	1.40	0.92
74	1.19	3.84	3.00	2.44	1.62	1.08	105	1.70	3.36	2.64	2.20	1.40	0.91
75	1.21	3.84	3.00	2.44	1.62	1.07	106	1.71	3.36	2.64	2.18	1.40	....
76	1.23	3.84	2.94	2.44	1.62	1.06	107	1.73	3.36	2.64	2.16	1.40	....
77	1.24	3.84	2.88	2.44	1.62	1.05	108	1.74	3.36	2.64	2.16	1.40	....
78	1.26	3.84	2.88	2.44	1.60	1.04	109	1.76	3.36	2.64	2.16	1.38	....
79	1.27	3.72	2.88	2.44	1.58	1.03	110	1.78	3.36	2.64	2.10	1.38	....
80	1.29	3.72	2.88	2.40	1.56	1.03	111	1.79	3.36	2.64	2.08	1.38	....
81	1.31	3.72	2.88	2.40	1.56	1.02	112	1.81	3.36	2.60	2.08	1.36	....
82	1.32	3.72	2.88	2.40	1.54	1.02	113	1.82	3.36	2.58	2.08	1.36	....
83	1.34	3.72	2.88	2.40	1.54	1.01	114	1.84	3.30	2.58	2.08	1.36	....
84	1.35	3.60	2.82	2.40	1.54	1.00	115	1.86	3.24	2.58	2.08	1.36	....
85	1.37	3.60	2.82	2.40	1.54	1.00	116	1.87	3.24	2.58	2.08	1.36	....
86	1.39	3.60	2.76	2.40	1.54	1.00	117	1.89	3.24	2.58	2.08	1.34	....
87	1.40	3.60	2.76	2.40	1.54	1.00	118	1.91	3.24	2.58	2.04	1.34	....
88	1.42	3.60	2.76	2.34	1.52	1.00	119	1.92	3.24	2.52	2.04	1.34	....
89	1.44	3.60	2.76	2.32	1.52	1.00	120	1.94	3.24	2.52	2.04	1.34	....
90	1.45	3.60	2.76	2.32	1.52	1.00	121	1.95	3.24	2.52	2.04	1.32	....
91	1.47	3.60	2.76	2.32	1.50	1.00	122	1.97	3.24	2.52	2.00	1.32	....
92	1.48	3.60	2.70	2.32	1.50	0.99	123	1.98	3.24	2.52	2.00	1.32	....
93	1.50	3.60	2.70	2.32	1.48	0.98	124	2.00	3.24	2.52	2.00	1.30	....

TABLE 3.—(*Continued*)

Serial No.	Average number per year	Duration, in Minutes				Serial No.	Average number per year	Duration, in Minutes			Serial No.	Average number per year	Duration, in Minutes	
		5	10	15	30			5	10	30			10	30
125	2.01	3.24	2.50	2.00	1.30	156	2.51	3.00	2.28	1.12	187	3.02	2.04	1.00
126	2.03	3.20	2.48	2.00	1.30	157	2.53	3.00	2.28	1.12	188	3.04	2.04	1.00
127	2.05	3.12	2.46	2.00	1.28	158	2.55	3.00	2.28	1.10	189	3.06	2.04	1.00
128	2.07	3.12	2.46	2.00	1.28	159	2.57	3.00	2.22	1.10	190	3.07	2.04	1.00
129	2.08	3.12	2.46	2.00	1.28	160	2.58	3.00	2.22	1.10	191	3.08	2.00	0.98
130	2.10	3.12	2.46	2.00	1.26	161	2.59	3.00	2.22	1.10	192	3.10	2.00	0.98
131	2.12	3.12	2.46	2.00	1.26	162	2.61	3.00	2.22	1.10	193	3.11	2.00	0.96
132	2.13	3.00	2.40	2.00	1.24	163	2.63	3.00	2.20	1.10	194	3.13	2.00	0.94
133	2.15	3.00	2.40	2.00	1.24	164	2.65	3.00	2.16	1.10	195	3.15	2.00	0.90
134	2.16	3.00	2.40	2.00	1.24	165	2.66	3.00	2.16	1.09	196	3.16	2.00	....
135	2.18	3.00	2.40	1.96	1.22	166	2.67	3.00	2.16	1.08	197	3.18	1.98	....
136	2.19	3.00	2.40	1.96	1.22	167	2.69	....	2.16	1.08	198	3.20	1.98	....
137	2.21	3.00	2.40	1.96	1.22	168	2.71	....	2.16	1.08	199	3.21	1.98	....
138	2.23	3.00	2.40	1.96	1.22	169	2.73	....	2.10	1.08	200	3.23	1.98	....
139	2.24	3.00	2.40	1.95	1.20	170	2.75	....	2.10	1.08	201	3.24	1.98	....
140	2.26	3.00	2.40	1.92	1.20	171	2.76	....	2.10	1.06	202	3.26	1.92	....
141	2.28	3.00	2.40	1.92	1.20	172	2.77	....	2.10	1.06	203	3.27	1.92	....
142	2.29	3.00	2.40	1.92	1.20	173	2.79	....	2.10	1.06				
143	2.31	3.00	2.40	1.92	1.20	174	2.81	....	2.10	1.06				
144	2.33	3.00	2.40	1.92	1.20	175	2.83	....	2.10	1.06				
145	2.34	3.00	2.40	1.92	1.20	176	2.84	....	2.10	1.04				
146	2.36	3.00	2.34	1.92	1.20	177	2.85	....	2.10	1.04				
147	2.37	3.00	2.28	1.92	1.20	178	2.87	....	2.10	1.04				
148	2.39	3.00	2.28	....	1.18	179	2.89	....	2.10	1.02				
149	2.41	3.00	2.28	....	1.16	180	2.91	....	2.10	1.00				
150	2.42	3.00	2.28	....	1.16	181	2.92	....	2.10	1.00				
151	2.44	3.00	2.28	....	1.16	182	2.93	....	2.10	1.00				
152	2.45	3.00	2.28	....	1.16	183	2.95	....	2.04	1.00				
153	2.47	3.00	2.28	....	1.14	184	2.97	....	2.04	1.00				
154	2.48	3.00	2.28	....	1.14	185	2.99	....	2.04	1.00				
155	2.50	3.00	2.28	....	1.12	186	3.00	....	2.04	1.00				

The data in Table 3 may be treated in three ways:

1.—A relation may be obtained for the data taken along the horizontal lines showing a relation for each frequency between duration and rainfall intensity.

2.—A relation may be obtained for the data listed in vertical columns under each given duration. This will show a relation for each duration between the rainfall intensity and the frequency.

3.—A single relation may be obtained between the three factors of duration, rainfall intensity, and frequency, using all the data both horizontally and vertically in a single comprehensive calculation.

TABLE 4.—CONDENSED STORM DATA, CENTRAL PARK GAUGE, NEW YORK CITY, 1869 TO 1930, INCLUSIVE

Duration, in minutes	Serial No. Times per year Storm designation	62 1 1 yr	31 0.5 2 yr	12.4 0.2 5 yr	6.2 0.1 10 yr	2.48 0.04 25 yr	1.24 0.02 50 yr
5		4.08	5.04	6.36	6.98	8.31	9.09
10		3.06	3.66	5.06	5.28	7.59	8.86
15		2.56	3.12	4.09	4.78	7.23	8.33
30		1.72	2.14	2.74	3.22	5.23	5.44
60		1.10	1.33	1.70	2.12	2.90	3.24
120		....	....	1.06	1.29	1.62	1.81

It is more convenient for general use to obtain the first form of relationship between the duration and the rainfall intensity for the "1-year" and other storms. In the Borough of Manhattan at least the "10-year" storm would be used for sewer design, whereas for certain parts of the Borough of Queens, the formula or curve for a "2-year" storm would be satisfactory.

The second form of relationship between frequency and rainfall intensity will give for each duration an entire gamut of storms, including 1-year, 2-year, and others. The third form, combining into one formula a relation between the three variable factors of intensity, duration, and frequency, would graduate the data on a surface, whereas the previous forms graduate the factors on a series of lines without showing any regularity among the different lines. All three forms of relationship are exemplified in this paper.

RECIPROCAL FORMULA

In Greater New York there is a partiality for a reciprocal formula of the type:

$$R = \frac{A}{t + b} \quad \dots\dots\dots\dots\dots\dots (1)$$

giving the relation between t, the duration of the storm, in minutes, and R, the corresponding rainfall intensity, in inches per hour, equalled or exceeded once in a certain number, n, of years. Hence, Equation (1) would be the formula for a theoretical "n-year" storm. For a 1-year storm ($n = 1$) and, similarly, for other storms, A and b would have one set of fixed values, which

must be obtained for that storm. If done by trial this would be a laborious hit-and-miss method; but by the method of least squares the data given in Table 3 are sufficient. Calculations of A and b may be simplified by the following consideration: If the logarithms of Equation (1) are taken, it becomes,

$$\log R = \log A - \log (t + b) \qquad \text{(2)}$$

This equation is linear between $\log R$ and $\log (t + b)$. If logarithmic ruled paper is used the R and t data will plot as an irregular line or curve for each character of storm. If, however, to each value of t a certain fixed value for b is added, the effect is to straighten the curve. That value of b is used which will make the transformed line most nearly straight and, at the same time,

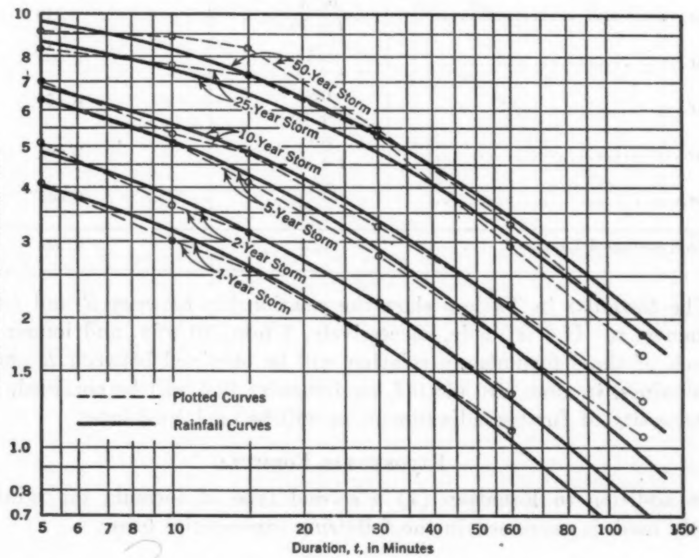

FIG. 1.—RAINFALL CURVES IN FORM, $R = \frac{A}{t + b}$. DATA FROM CENTRAL PARK GAUGE, NEW YORK, N. Y., 1869 TO 1930. FORMULAS FOR VARIOUS STORMS GIVEN IN TABLE 5.

plot to an angle of 45° with the axes. The graphs for Equation (1) are shown on Fig. 1, using the data from Table 3. The transformed lines are not indicated.

Now that b is known, A is computed from Equation (2) by the method of least squares as illustrated for a more complicated case in the Appendix.

Formulas of the type given in Equation (1) for the various storms are plotted in Fig. 1 and listed in Table 5. Comparisons are then made between the observed results and the results obtained from the formulas. The average difference between the observed intensity and the result from Equation (1) equals 0.182 in. per hr for the 10-year storm. Similar average differences for the various characters of storms have been computed as shown in Table 5.

The final average difference computed as described in the Appendix, is 0.303 which may be taken as a measure of the closeness with which formulas of the type of Equation (1), as illustrated in Fig. 1, fit the observations. This diagram shows that expressions similar to Equation (1) cannot be made to fit the data as accurately as is desired.

TABLE 5.—VARIATION OF RAINFALL CURVES BY RECIPROCAL FORMULA FROM OBSERVED DATA

Storms	Values for Rate R, Equation (1)	Average difference from observed data, in inches per hour
1-year	$\frac{76.4}{t + 14}$	0.075
2-year	$\frac{96.3}{t + 15}$	0.140
5-year	$\frac{128}{t + 15}$	0.099
10-year	$\frac{168}{t + 20}$	0.182
25-year	$\frac{261}{t + 25}$	0.404
50-year	$\frac{291}{t + 25}$	0.563
Final average difference	......	0.303

The formulas in Table 5 show the relationship between R and t for each frequency, n. If t is made, successively, 5 min, 10 min, and longer periods, in each of these formulas, a relation will be obtained between R and n. If these values, in turn, are plotted, an irregular line will be obtained, showing the necessity of further adjustment, as will be explained later.

EXPONENTAL FORMULA

In addition to Equation (1) a second type of formula for rainfall frequently used, is expressed in the following exponential form:

$$R = \frac{B}{t^c} \quad \text{.............................(3)}$$

in which, B and c are constants, having one set of values for a 1-year storm and a different set for a 2-year and other storms. Taking the log of Equation (3),

$$\log R = \log B - c \log t \quad \text{......................(4)}$$

which is a straight line between $\log R$ and $\log t$. This line may be obtained directly, whereas in Equation (1) a constant value had to be added to t to yield an approximate straight line. The tangent of the slope of the transformed lines represented by Equation (1) is the same for all storms, when their graphs are plotted with similar scales in both directions on logarithmic paper for the 1-year and other storms, all making an angle of 45° to the "log t" line. In Equation (4) the tangent of the slope is c, which differs for various storms. Therefore, the lines representing such storms on logarithmic paper,

Fig. 2, will not be parallel. In this diagram, also, values of B and c have been determined by the method of least squares as given in the Appendix. The

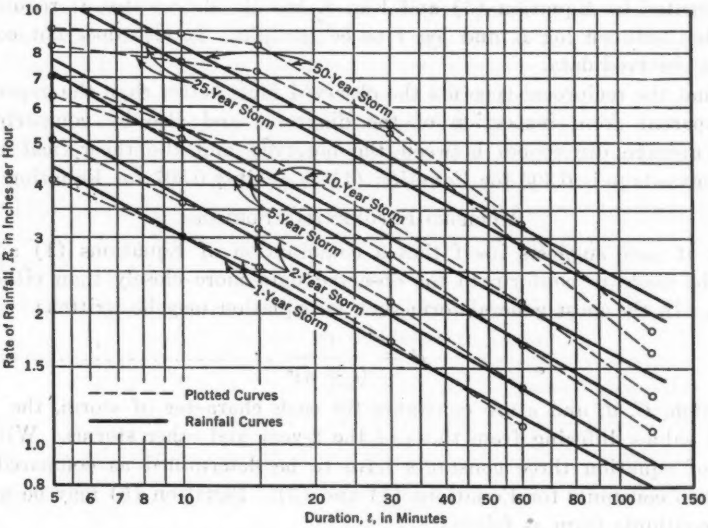

FIG. 2.—RAINFALL CURVES IN FORM, $R = \frac{B}{t^c}$. DATA FROM CENTRAL PARK GAUGE, 1869 TO 1930. FORMULAS FOR VARIOUS STORMS GIVEN IN TABLE 6.

resulting forms of Equation (3) and the average deviations for the different storms are given in Table 6.

TABLE 6.—VARIATION OF RAINFALL CURVES BY EXPONENTIAL FORMULA FROM OBSERVED DATA

Storm	Values for Rate R, Equation (3)	Average difference from observed data, in inches per hour
1-year........................	$\frac{10.11}{t^{0.529}}$	0.128
2-year........................	$\frac{12.38}{t^{0.53}}$	0.144
5-year........................	$\frac{18.1}{t^{0.576}}$	0.376
10-year........................	$\frac{18.1}{t^{0.53}}$	0.370
25-year........................	$\frac{24.8}{t^{0.526}}$	1.27
50-year........................	$\frac{28}{t^{0.53}}$	1.41
Final average difference..........	....	0.807

The first form of rainfall formula, the reciprocal type illustrated by Equation (1), has the defect that it requires the straight line between log R and log $(t + b)$ to be at a definite inclination of 45° downward to the right for all characters of storm, and, therefore, may not be made to conform to

the observed data as closely as desired. This was actually found to be the case, as may be seen in Fig. 1. The second, or exponential, type, as represented by Equation (3) and Fig. 2, has the defect that it requires the relation between log R and log t to be straight. It also does not conform to the observed data.

That the reciprocal type fits the observed facts better than the exponential is apparent from inspection of the diagrams and also by comparing the final average differences between the observed and the theoretical results. The advantage is 0.303 for Equation (1) as against 0.807 for Equation (3).

Modified Exponential Formula

It at once suggests itself that a combination of Equations (1) and (3) may be made to conform to the observed data more closely than either one alone. In the most general form such an equation may be written:

$$R = \frac{C}{(t + d)^e} \quad \text{.......................... (5)}$$

in which, C, d, and e are constants for each character of storm, the 1-year storm values differing from those of the 2-year and other storms. With this type of equation three constants have to be determined as compared with only two constants for Equations (1) and (3). Equation (5) may be written in logarithmic form as follows:

$$\log R = \log C - e \log (t + d) \quad \text{.................. (6)}$$

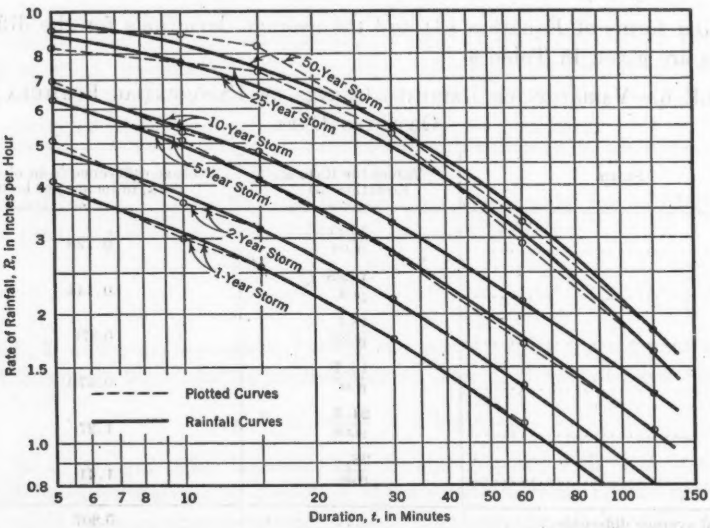

Fig. 3.—Rainfall Curves in Form, $R = \frac{C}{(t + d)^e}$. Data from Central Park Gauge, 1869 to 1930. Formulas for Various Storms Given in Table 7.

This is not linear between log R and log t, but is linear between log R and log $(t + d)$. Therefore, if the various observed values are plotted on logarithmic paper (Fig. 3), the lines connecting them will be irregularly bent

the same as in Fig. 1. When this bent line is transformed by adding a constant value, d, to each value of t, keeping the corresponding R the same, a differently bent line will be obtained. If the bending is increased, smaller values for d should be tried, and conversely until almost a straight line is obtained. This process is the same as was described in developing Fig. 1. The proper values of d which will make the transformed line almost straight can then be readily interpolated and checked by plotting.

After d has been obtained in this manner, e and C are computed by the method of least squares, as shown in detail for this case in the Appendix. The appropriate formulas for the various storm classifications and the resultant deviation of the plotted data from the curves for R (Fig. 3), are given in Table 7.

TABLE 7.—VARIATION OF RAINFALL CURVES BY MODIFIED EXPONENTIAL FORMULA FROM OBSERVED DATA

Storm	Values for Rate R, Equation (5)	Average difference from observed data, in inches
1-year..........................	$\frac{38.85}{(t+10)^{0.842}}$	0.057
2-year..........................	$\frac{41.62}{(t+9)^{0.813}}$	0.096
5-year..........................	$\frac{60.53}{(t+10)^{0.838}}$	0.061
10-year..........................	$\frac{63.75}{(t+12)^{0.795}}$	0.155
25-year..........................	$\frac{1\,468}{(t+40)^{1.342}}$	0.329
50-year..........................	$\frac{4\,201}{(t+50)^{1.513}}$	0.428
Final average difference..........	....	0.235

About 1921 the Bureau of Sewers of the City of New York developed various formulas by a trial-and-error method from the rainfall data, which formulas were plotted on ordinary cross-section paper. These formulas were

TABLE 8.—COMPARISON OF VARIOUS FORMULAS FOR RAINFALL, WITH OBSERVED DATA

Types of formulas	Two-Year Storm		Ten-Year Storm	
	Formulas	Average difference from observed data, in inches per hour	Formulas	Average difference from observed data, in inches per hour
Old formulas..........................	$\frac{120}{t+20}$	0.285	$\frac{150}{t+15}$	0.387
Reciprocal formulas (Fig. 1)........	$\frac{96.3}{t+15}$	0.140	$\frac{168}{t+20}$	0.182
Exponential formulas (Fig. 2).......	$\frac{12.38}{t^{0.53}}$	0.144	$\frac{18.1}{t^{0.53}}$	0.370
Modified exponential formulas (Fig. 3)	$\frac{41.62}{(t+9)^{0.813}}$	0.096	$\frac{63.75}{(t+12)^{0.795}}$	0.155

of the type given in Equation (1). It is interesting to give comparative results between these "old formulas" and the various formulas presented in this paper, as shown in Table 8. The average differences from the observed data are smallest for the formulas in Table 7, plotted in Fig. 3, and largest for the old formulas.

Relation Between Rainfall Intensity and Frequency

If the observed data in Table 3 are plotted for each duration given in the vertical columns, irregular lines will be obtained, showing the relation between the rainfall intensity, R, and the frequency, n, for the duration given. It is evident that there is as much justification for seeking order in the relation between R and n for each duration, t, as between R and t for each frequency, n, or character of storm that has been obtained in Figs. 1, 2, and 3. Studies have been made to obtain such relationship from Table 3.

The equation connecting R, rainfall intensity, and n, average number of times per year which it occurs or is exceeded, is:

$$R\,n^h = k \qquad (7)$$

in which, h and k are constants for each duration, t. The various values of h and k are given in Fig. 4 with the curves and the observed data.

These formulas in Fig. 4 have been computed from the observed data by the method of least squares. The results will agree, in general, only approximately with those obtained from the formulas in Fig. 3.

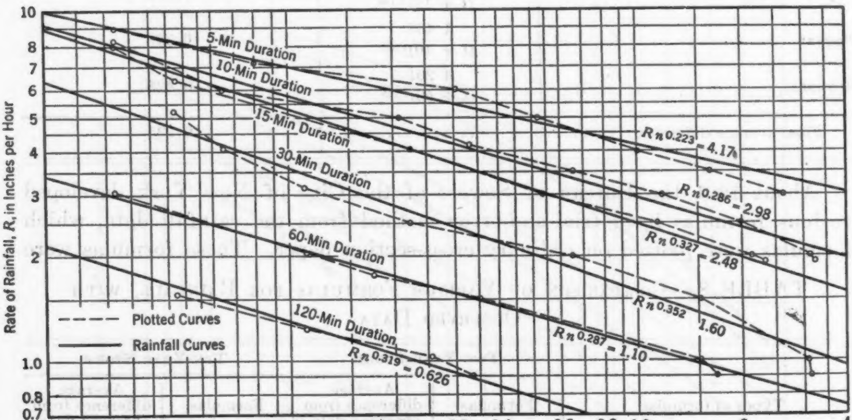

FIG. 4.—HEAVY RAINFALL FREQUENCIES IN FORM, $R^h{}_n = k$. DATA FROM CENTRAL PARK GAUGE, NEW YORK, N. Y., FROM 1869 TO 1930.

It is desirable to harmonize all the data given in Table 3 showing a relation between R, t, and n. An approximate relation between the three factors has been obtained, as follows:

$$R\,n^{0.3} = \frac{42.5}{(t+12)^{0.85}} \qquad (8)$$

The logical refinement leading to the development of Equation (8) may be unnecessary in practice. It would probably be expedient to use the set of formulas which best fit the observations even if they do not harmonize with one another. Accordingly, it is recommended that the formulas in Table 7, and Fig. 3 be used in sewer investigations and design.

Probability Method

Rainfall records note certain facts which have already occurred. Like history they teach by example. If the conditions continue with little alteration, it is assumed that the past records may be used to anticipate what may occur in the future. Any regularity which may be perceived in past events may be assumed to recur in the future.

Various expressions have been given in this paper which tend to show such regularity by a method known as "curve fitting." Other methods can also be used in interpreting those records for future guidance. The best known are those used in biometrics and vital statistics, as developed by Karl Pearson and his school. They are offshoots of the theory of probability.

The duration curve is sometimes called the integral curve and gives the relative frequency that a certain magnitude has been equalled or exceeded. The magnitude may refer to average monthly or annual stream flow, total annual rainfall, or rainfall intensity for a 15-min duration. The frequency curve gives only the frequency of occurrence of a definite magnitude, showing, for instance, how many times an annual rainfall of 40 to 45 in. may be expected to occur in 100 years. The integral or duration formula is obtained from the frequency formula. Its expression is generally very complicated.

Various types of frequency curves or equations have been suggested. "Types I and III" appear to fit engineering data best. Type I applies to a range of records which are skewed and have both a finite minimum and a finite maximum value. Type III is used for records having a finite value in one direction only. From the manner in which the variations of the magnitude from the average occur in the recorded data the probable skew value is obtained by certain mathematical processes.

This probability method, applied to the data in Tables 2 and 3, shows that the Type III curve fits the data best. The coefficient of skew appears to vary from 1.56 to 3.02, with an average value of 2.22. The curves in Fig. 5 have been prepared for durations of 5, 10, 15, 30, 60 and 120 min, based on the Type III curve with a uniform coefficient of skew, $C\ S = 2.22$. From Fig. 5 it appears that once in 50 years a rainfall intensity of 7.40 in. per hr and more may occur for a 10-min duration, 4.55 in. per hr and more for 30 min; and 2.80 in. per hr and more for 60 min.

A comparison between the results obtained from Figs. 3 and 5 is given in Table 9. An inspection of Table 9 indicates that the modified exponential method gives results more in accord with the actual data than the probability method.

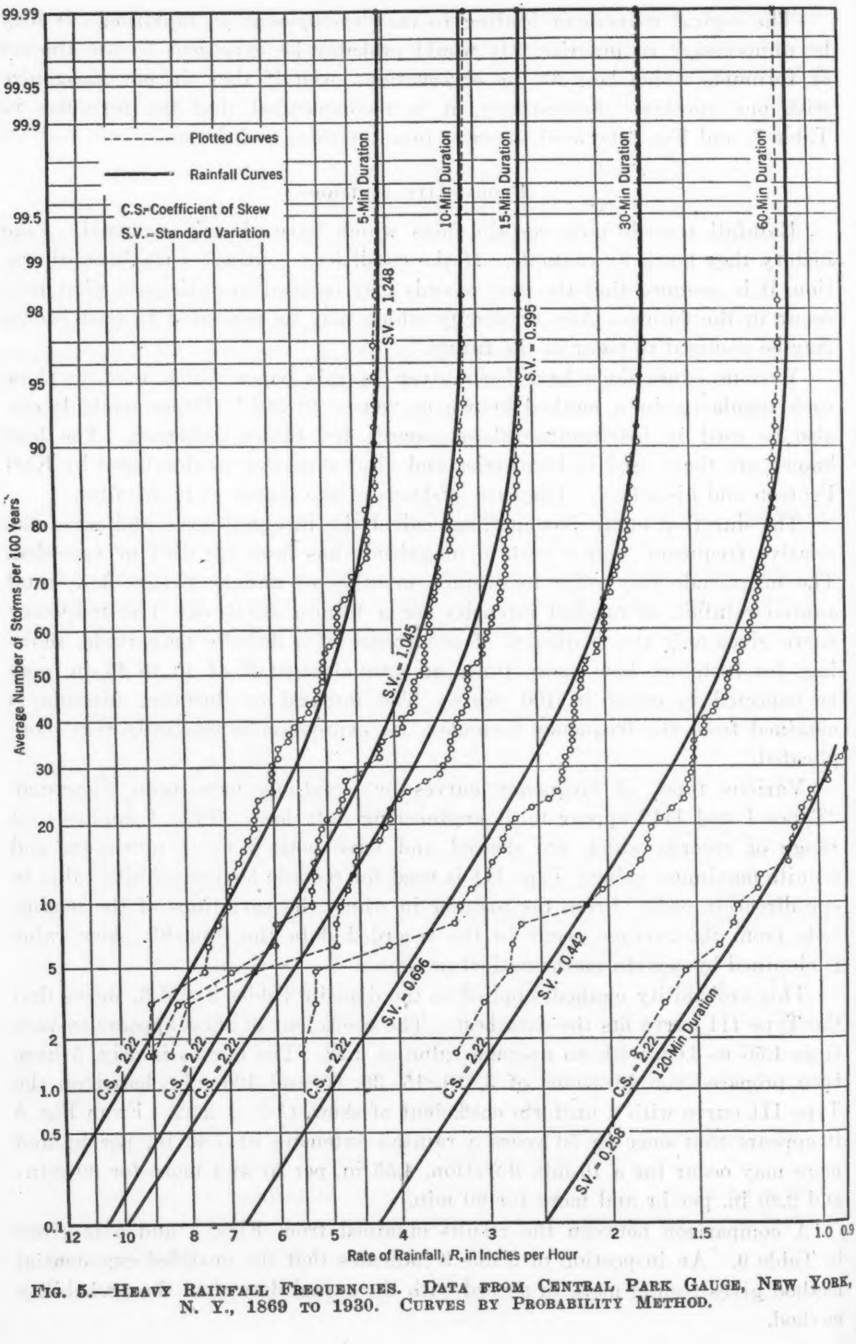

FIG. 5.—HEAVY RAINFALL FREQUENCIES. DATA FROM CENTRAL PARK GAUGE, NEW YORK, N. Y., 1869 TO 1930. CURVES BY PROBABILITY METHOD.

TABLE 9.—RAINFALL INTENSITIES BY MODIFIED EXPONENTIAL AND PROBABILITY METHODS COMPARED WITH ACTUAL INTENSITIES

Duration, in minutes	Frequency, in years	n	Modified exponential method, Fig. 3, in inches per hour	Probability method, Fig. 5, in inches per hour	Actual intensities, in inches per hour
10	Once in 50.	0.02	8.20	7.40	8.86
30	Once in 50.	0.02	5.50	4.55	5.44
60	Once in 50.	0.02	3.40	2.80	3.24
10	Once in 10.	0.10	5.40	5.50	5.28
30	Once in 10.	0.10	3.25	3.38	3.22
60	Once in 10.	0.10	2.10	2.12	2.12
10	Once in 2.	0.50	3.79	3.81	3.66
30	Once in 2.	0.50	2.12	2.20	2.14
60	Once in 2.	0.50	1.33	1.38	1.33

CONCLUSION

Various types of formulas have been developed showing the relations between the rainfall intensities, durations, and frequencies of occurrence. The results, however, do not equally fit the recorded storms, and the single unified formula, Equation (8), does not appear to be sufficiently close to the observations to warrant its use for all kinds of storms. The six formulas given in Table 7 and plotted in Fig. 3 answer most accurately the requirements of the engineer, either for computing storm sewers or for making investigations of the flooding of buildings or of underground subways in New York City. It is also believed that this paper gives in ample detail the procedure to be used in obtaining similar formulas for locations other than New York City.

ACKNOWLEDGMENT

The paper was prepared in the course of work in the Sewer and Drainage Sub-Division of the Designing Office of the Board of Transportation, New York City. The collection of the data and the laborious calculations were made by William A. O'Leary, Assoc. M. Am. Soc. C. E. Acknowledgment is made to C. E. Conover, Division Engineer of the Designs Division, A. I. Raisman, Chief Designing Engineer, Members, Am. Soc. C. E., and Robert Ridgway, Past-President, Am. Soc. C. E., Consulting Engineer, of the Board of Transportation, all of whom made the work of preparing this paper possible.

APPENDIX

DERIVATION OF FORMULA FOR 10-YEAR STORM IN THE FORM, $R = \dfrac{C}{(t+d)^e}$

This Appendix explains the calculation of the modified exponential formula, Equation (5), for a 10-year storm; the calculations of the reciprocal formula, Equation (1), and the exponential formula, Equation (2), are similar. The data from Table 3 for a 10-year storm are plotted on logarithmic ruled paper shown as a broken line. By adding, however, a constant, 12, to t the graph will become very nearly straight. With Equation (5), the slope

of the line is not restricted to 45° as required by Equation (1). The equation of this line may be found by the method of least squares. In this case: $R = \frac{C}{(t + 12)^e}$. Taking logarithms of both sides,

$$\log R = \log C - e \log (t + 12)$$

and using observed values of R and t,

$$\log R - \log C + e \log (t + 12) = D$$

in which, D is a small number. Then,

$$\Sigma [\log R - \log C + e \log (t + 12)]^2 = \Sigma D^2 = M$$

For M to be a minimum,

$$\frac{\partial M}{\partial \log C} = 0$$

$$\frac{\partial M}{\partial e} = 0$$

and,

$$\frac{\partial M}{\partial \log C} = -2 \Sigma [\log R - \log C + e \log (t + 12)] = 0$$

that is,

$$\Sigma \log R = n \log C - e \Sigma \log (t + 12) \ldots. \text{(1st Normal Equation)}$$

in which, n is the number of observations, which is equal to 6 in Table 10. Also,

$$\frac{\partial M}{\partial e} = 2 \Sigma \left\{ [\log R - \log C + e \log (t + 12)] \times \log (t + 12) \right\} = 0$$

or,

$$\Sigma [\log R \times \log (t + 12)] = \log C \Sigma (t + 12)$$
$$- e \Sigma [\log (t + 12)]^2 \ldots\ldots\ldots \text{(2d Normal Equation)}$$

TABLE 10.—CALCULATION OF FACTORS IN NORMAL EQUATIONS.

t (1)	$t + 12$ (2)	$\log (t + 12)$ (3)	$[\log (t + 12)]^2$ (4)	Observed R (5)	$\log R$ (6)	$\log R \times \log (t + 12)$ (7)	Computed R (8)	Difference (9)	Difference[2] (10)
5........	17	1.2304	1.5139	6.98	0.8439	1.0383	6.69	−0.29	0.0841
10........	22	1.3424	1.8020	5.28	0.7226	0.9700	5.46	+0.18	0.0324
15........	27	1.4314	2.0489	4.78	0.6794	0.9725	4.62	−0.16	0.0256
30........	42	1.6232	2.6348	3.22	0.5079	0.8244	3.26	+0.04	0.0016
60........	72	1.8573	3.4495	2.12	0.3263	0.6060	2.13	+0.01	0.0001
120........	132	2.1206	4.4969	1.29	0.1106	0.2345	1.32	+0.03	0.0009
Sums......	....	9.6053	15.9460	......	3.1907	4.6457	......	......	0.1447
Average difference	....	....	....	......	......	....	......	......	0.155

The data and computations from the observed values of t are listed most conveniently in the form given in Table 10. From this is found:

$$\text{Average difference} = \sqrt{\frac{0.1447}{6}} = 0.155$$

Completing the normal equations for the summations in Table 10,

$$3.1907 = 6 \log C - e \times 9.6053$$

and,

$$4.6457 = 9.6053 \log C - e \times 15.9460$$

Solving, $e = 0.795$ and $C = 63.75$. Therefore,

$$R = \frac{63.75}{(t + 12)^{0.795}}$$

in which, R = rate of rainfall, in inches per hour, and t = duration, in minutes.

The rate and [illegible]

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Combining the [illegible]

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE

BY NORMAN B. GREEN,[1] ESQ.

SYNOPSIS

A method of analyzing multiple-story, steel-frame buildings to resist earthquakes, is presented in this paper. The theory is based on a flexible first story, without attempting to draw definite conclusions regarding the relative practicability of this type of construction. The conclusions to be drawn from the evidence presented is summarized at the end of the paper.

The problem of constructing high buildings of twenty or thirty stories so that they shall be capable of resisting earthquakes, has brought forth the suggestion that they be provided with a flexible story, upon which the structure shall swing. The original idea was that the earth motion would be absorbed in this flexible story, so that the upper part of the building need not be designed to resist a lateral earthquake force. Actually, such a structure is an elastic vibrating system and its maximum lateral deflection must be determined. Many high buildings on the Pacific Coast are stated to be of this flexible construction. However, no one knows how they will actually behave during an earthquake, since with one or two exceptions no attempt has been made at a dynamic design. In the exceptional cases, recourse was had to a dynamic model tested on the shaking-table. If this type of construction is to be of practical value, it is imperative that some convenient design method be developed, since every engineer does not have access to a shaking-table and, moreover, the experimental method is both costly and time-consuming.

On a first approach to the problem it would appear that, by reason of the great irregularity of the earth motion (which is demonstrated by nearly any seismograph record), a mathematical solution is impossible. The designer, however, is not concerned with this motion of the earth as recorded by a seismograph, but he is interested in the acceleration and period of accelera-

NOTE.—Discussion on this paper will be closed in May, 1934, *Proceedings*.
[1] Structural Engr., San Francisco, Calif.

tion (that is, the time interval between two maxima of acceleration), since it is this change of velocity that provides the impulse to deflect an engineering structure and to set it in vibration. It is only recently that instruments have been available for measuring this acceleration, and an examination of the few available records of this kind taken during an earthquake show clearly that an accelerograph gives a much more regular swing than a seismograph recording the same earthquake. This regularity of swing and the fact of a nearly constant period of acceleration, can be best demonstrated by an examination of the accelerogram shown in Fig. 1, which is

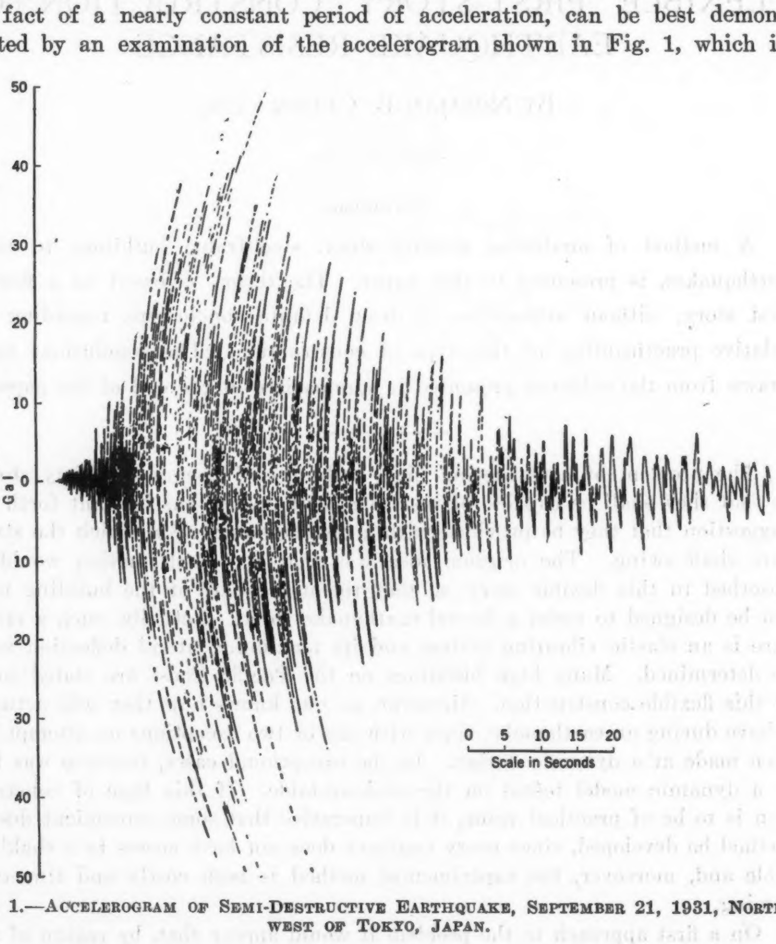

FIG. 1.—ACCELEROGRAM OF SEMI-DESTRUCTIVE EARTHQUAKE, SEPTEMBER 21, 1931, NORTHWEST OF TOKYO, JAPAN.

the record of a semi-destructive earthquake that occurred near Tokyo, Japan, in September, 1931.

The late Dr. K. Suyehiro concluded[2] that in all earthquakes the period of acceleration in a particular locality is confined within a very narrow range and that the motions of an earthquake that cause the predominant accelera-

[2] *Proceedings*, Am. Soc. C. E., May, 1932, Part 2.

tions are due to the habitual motions peculiar to the ground. It was found that the prevailing periods for earthquakes in Japan were generally from 0.7 to 0.9 sec on alluvial ground and from 0.3 to 0.4 sec on well-consolidated ground.

A further examination of Fig. 1 will show that the acceleration varies directly with the time. This fact is particularly apparent in the right-hand part of the diagram. It is also apparent that the maximum acceleration is by no means uniform, since a full swing may be immediately followed by a much shorter swing, or by one that is unbalanced; in fact, any combination seems to be possible.

With these facts established, it is feasible to construct an artificial acceleration diagram and to compute what effect the corresponding earth motion will have upon the relatively simple type of elastic structure, as represented by a building with a flexible base. In order that the method shall be general in its application, it must embrace the case of an irregularly varying maximum acceleration as well as the possibility of a varying period of acceleration. This requirement necessitates that the equations of motion be written separately for each swing and that they be solved successively, using the terminal velocity and deflection for one swing as the initial ones for the succeeding swing. This is the method that will now be developed. It may be visualized as a kind of mathematical shaking-table, in which the artificial acceleration diagram has its counterpart in the cam, which is cut so as to impart to the table a motion that simulates a given earthquake.

Consider a weight that rests upon rollers and is connected by two springs to Supports A and B, as indicated in Fig. 2. Assume that this weight is

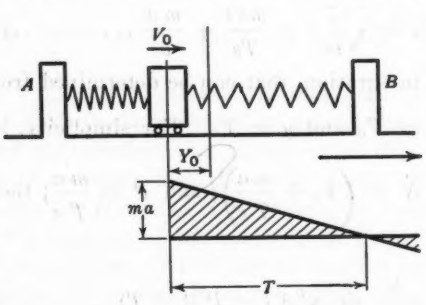

FIG. 2.

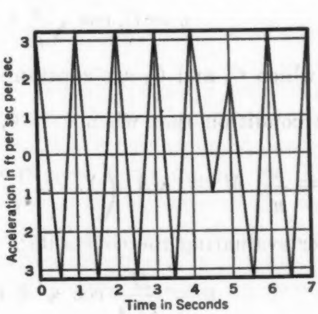

FIG. 3.—ACCELERATION DIAGRAM.

given an initial deflection, Y_0, and velocity, V_0, and that the axis of coordinates is taken through the position of static equilibrium. Assume furthermore that Supports A and B are accelerated toward the right, starting with an acceleration, a, which decreases as the ordinate of a straight line, to a zero acceleration at the expiration of a time, T, and continues a negative acceleration, as indicated in the lower diagram of Fig. 2. The ensuing motion of the supports, A and B, represents one swing of the artificial earthquake,

and the motion of the weight can be determined, using the following notation:

W = weight of vibrating body, in pounds;
g = acceleration of gravity = 32.2 ft per sec per sec;
m = mass of vibrating body $= \frac{W}{g}$
a = initial acceleration of ground at the beginning of the swing, in feet per second per second;
e = elastic constant of vibrating system $= \frac{\text{Lateral force}}{\text{Deflection}}$;
t = independent variable, time, in seconds;
T = elapsed time from initial ground acceleration to zero acceleration;
y = deflection of the body from its position at rest, in feet;
Y_0 = initial deflection at $t = 0$;
V_0 = initial velocity at $t = 0$, in feet per second;
f = maximum deflection during one swing;
t_m = elapsed time from $t = 0$ to the maximum deflection;
v = velocity at any time, t;
v_n = velocity at the end of the nth swing;
y_n = deflection at the end of the nth swing; and,
t_n = elapsed time from the beginning to the end of the nth swing.

If positive forces, deflections, accelerations, and velocities are each taken in the direction of the initial force, $m\ a$, the differential equation of motion is expressed as:

$$m \frac{d^2y}{dt^2} = m\,a \left(1 - \frac{t}{T}\right) - e\,y \quad \dots\dots\dots\dots\dots\dots (1)$$

and the general solution of this linear equation is:

$$y = C_1 \cos \sqrt{\frac{e}{m}}\, t + C_2 \sin \sqrt{\frac{e}{m}}\, t - \frac{m a t}{T e} + \frac{m\,a}{e} \quad \dots\dots\dots\dots (2)$$

in which C_1 and C_2 are constants of integration, that can be determined from the condition that when $t = 0$, $\frac{dy}{dt} = V_0$ and $y = Y_0$. For simplicity, let $A = \frac{e}{m}$; $M = \sqrt{A}\left(Y_0 - \frac{m\,a}{e}\right)$; $N = \left(V_0 + \frac{m\,a}{T\,e}\right)$; and $P = \frac{m\,a}{T\,e}$; then, after evaluating the constants:

$$y = \frac{M}{\sqrt{A}} \cos \sqrt{A}\ t + \frac{N}{\sqrt{A}} \sin \sqrt{A}\, t - P\,(t - T) \quad \dots\dots\dots (3)$$

and by differentiating again;

$$\frac{dy}{dt} = v = -M \sin \sqrt{A}\ t + N \cos \sqrt{A}\ t - P \quad \dots\dots\dots (4)$$

The value of t_m may be obtained by setting Equation (4) equal to zero and solving for t, whence:

$$t_m = \frac{1}{\sqrt{A}} \tan^{-1}\left(\frac{M\,N \pm P\sqrt{N^2 + M^2 - P^2}}{M^2 - P^2}\right) \dots\dots\dots (5)$$

and if the quantity in the parenthesis is set equal to k:

$$t_m = \frac{1}{\sqrt{A}} \tan^{-1} k \text{ ....(6)}$$

The maximum deflection is obtained by substituting the value, t_m, into Equation (3), whence:

$$f = \frac{M}{\sqrt{A}} \cos(\tan^{-1} k) + \frac{N}{\sqrt{A}} \sin(\tan^{-1} k) - \frac{P}{\sqrt{A}} (\tan^{-1} k) + PT \text{ ..(7)}$$

As a special case of Equation (3), suppose that $a = 0$ and $V_0 = 0$, in which case this equation reduces to:

$$y = Y_0 \cos \sqrt{A}\, t \text{ ....(8)}$$

which represents simple harmonic motion. By equating the first derivative to zero and solving for t:

$$t = 2\pi \sqrt{\frac{m}{e}} \text{ ....(9)}$$

which is the period of free vibration of the structure.

As another special case let T equal infinity, which simply means that the acceleration, a, is constant and that Equation (7) then reduces to:

$$f = -\frac{m\,a}{e} (\cos \tan^{-1} 0 - 1) = -\frac{m\,a}{e} (\pm\ 1 - 1) \text{ ....(10)}$$

The minus sign must be used to get the maximum deflection, since the plus sign gives $f = 0$, which is the minimum deflection. Therefore:

$$f = \frac{2\,m\,a}{e} \text{ ....(11)}$$

but $\frac{m\,a}{e}$ represents the static deflection produced by a constant acceleration, a, and it is apparent, therefore, that with a suddenly applied acceleration which is in the nature of an impact, the maximum deflection produced is twice the static deflection. In order that this deflection can be developed, however, it is necessary that the earth motion be of sufficient duratioin in one direction; but since the effect is entirely independent of the stiffness of the structure (that is, of the value of e), a very stiff structure would require a very short motion. While the earthquake acceleration, of course, is not uniform, it may be considered so for a very short period of time or when the swing is very long.

A glance at Fig. 1 will show that the assumption of an instantly applied acceleration may not be so far from the truth, since the accelerogram starts its major motion with a long swing. This same abrupt start of the major motion is indicated very clearly on many seismograms. To illustrate the sudden start of a violent earthquake in the epicentral region, Dr. Suyehiro gives the evidence of an eye-witness that copper coins in a tin can with a lid, threw off the lid and leaped out, leaving the can standing, at the instant the earthquake began. This effect would require a downward acceleration greater than gravity, with a motion of several inches of amplitude.

The evidence of eye-witnesses to the Long Beach, Calif., earthquake in 1933 also describes a very abrupt beginning of the primary shock. An examination of the California Garage, in Long Beach, which is a three-story reinforced concrete structure, gives clear evidence that the shock began as a sudden jolt or jerk toward the southeast. The columns were shattered at the top in compression on the southeast side, with a corresponding tension crack near the floor on the same side. On the opposite side there was a slight spalling at the floor line and a tension crack near the ceiling. In other words, this injury indicates destructive deformation in one direction only, which may be attributed to a heavy impact effect in the direction of the first swing.

It has been assumed in the foregoing analysis that the entire vibrating mass is concentrated at the point of elastic support, and that the mass is rigid. As regards the first of these assumptions, it is true that the mass of the building structure towers far above the base columns; however, the only uncertainty that this fact introduces, arises from the added horizontal motion that the upper part of the structure may sustain, due to tipping as a unit on the base columns. Calculation shows that for the building presently to be described, this horizontal motion of the top, due to direct column stress, is only about 2% that due to column flexure. The second assumption cannot be true, of course, for an actual building; however, it is the relative stiffness that is of importance and the upper part of the structure can be made very rigid relative to the base, by the use of concrete bracing walls or some other equally efficient construction.

By the use of Equations (3), (4), (6), and (7), the motion of the elastic structure can be completely determined for any number of swings and for any variation of the maximum acceleration and the period of acceleration. For the first swing, Y_0 and V_0 are zero and, having determined the values of Y_1 and V_1, these are used with reversed sign as Y_0 and V_0 for the second swing; the process being repeated indefinitely. This step-by-step solution presents a clear picture of what is happening to the structure and enables the designer to vary the nature of the earth motion as the computations progress, so as to subject the structure to the most unfavorable conditions. These computations may be carried out on a 10-in. slide-rule, but this should be carefully read to at least three significant figures, and four where possible, since the results involve small differences. It is necessary to work accurately, as the results for a given swing affect all succeeding swings.

The computation of f is rather laborious, but it need only be computed for those swings in which y_n is large, since one of these swings will contain the greatest deflection of the entire series. In this connection, it should also be noted that when y_n and v_n are of the same sign, y_n is actually the greatest deflection in that swing, since in this case t_m will be found to be greater than t_n, which simply means that the computed value of f occurs outside the range of the swing. The positive value of k should be used, care being exercised to select that value which gives the maximum deflection within the assumed range of the swing.

The design of the base columns, as well as the determination of their lateral deflection, requires the use of a formula for combined direct stress and bending since, with the large deflections attained, the eccentric moment is of great importance. For the case of a column fixed at the upper end and hinged at the base, the deflection, d, may be expressed as:

$$d = \frac{P}{W}\left(\frac{\tan a\,L}{a} - L\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which, L is the length of the column that carries a direct load, W, and a lateral, load, P, and $a = \sqrt{\frac{W}{E\,I}}$, in which, E and I are the modulus of elasticity and the moment of inertia, respectively. The value of d in Equation (12) is used in computing the stiffness factor and may also be used to compute the maximum column stress, S, from the familiar relation:

$$S = \frac{W}{A} + \frac{(PL + Wd)\,c}{I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which, A is the area of the column section, and c is the distance from the neutral axis to the extreme fiber.

It should be noted that the column deflection from Equation (12), is a function of the load on the column; but it is also directly proportional to the lateral force, P, so that e is a constant as has been assumed in the analysis.

Application of the Analysis

As an example of the method of computation, consider the case of a 20-story steel-frame building of the office type, with four 20-ft panels in one side and eight 18-ft panels along the other side. Suppose that the story height is 12 ft, except for the first story which has been taken as 24 ft. For a building of this type the weight per cubic foot will be about 17 lb when a 24-lb live load is included, and 15 lb when the live load is neglected. The former value has been used in designing the columns for vertical load, while in computing the mass of the structure the latter value has been used. The first-story columns are assumed as hinged at the base and with hinged second-floor beam connections, in order to increase their flexibility. These columns are designed to resist a wind pressure of 20 lb, with an over-stress of 50%, in which case the lateral deflection from Equation (12) is 0.194 ft and, using this deflection value, the stiffness factor, e, is 3 760 000 and the period of vibration from Equation (9) is 3.58 sec.

The acceleration diagram of Fig. 3 has been constructed, assuming a constant period of 1 sec and a maximum acceleration of 3.2 ft per sec per sec. This diagram represents a total elapsed time of 7.0 sec and contains seven complete vibrations or impulse cycles. It has been constructed so as to produce unfavorable conditions of deflection, but, nevertheless, it is believed to represent a perfectly possible situation.

Computations for this set of conditions are shown in Table 1. It will be noted that the maximum deflection of 0.445 ft occurs in the fourteenth swing,

and if this is compared with the maximum allowable deflection of 0.194, the columns are over-stressed by more than 100 per cent.

TABLE 1.—COMPUTATIONS

Swing	T	a	Initial Conditions		M	N	P	Maximum Deflection			Final Conditions		$\tan^{-1} k$
			Y_o	V_o				k	f	t_m	Y_n	V_n	
1...	0.25	3.2	0	0	—1.823	+4.150	4.150	......	......	.....	+0.120	—0.097	......
2...	0.25	3.2	—0.120	+0.097	—2.033	+4.247	4.150	......	......	.....	+0.082	+0.124	......
3...	0.25	3.2	+0.082	—0.124	—1.970	+4.026	4.150	......	......	.....	+0.009	—0.065	......
4...	0.25	3.2	+0.009	+0.065	—1.840	+4.215	4.150	......	......	.....	+0.143	—0.036	......
5...	0.25	3.2	—0.143	+0.036	—2.073	+4.186	4.150	......	......	.....	+0.043	+0.120	......
6...	0.25	3.2	—0.043	—0.120	—1.900	+4.030	4.150	......	......	.....	+0.037	—0.120	......
7...	0.25	3.2	—0.037	+0.120	—1.890	+4.270	4.150	......	......	.....	+0.145	+0.035	......
8...	0.25	3.2	—0.145	—0.035	—2.08	+4.115	4.150	......	......	.....	+0.011	+0.080	......
9...	0.38	3.2	—0.011	—0.080	—1.843	+2.650	2.730	......	......	.....	+0.162	+0.381	......
10...	0.167	1.0	—0.162	—0.381	—0.854	+1.562	1.943	......	......	.....	—0.272	—0.287	......
11...	0.193	2.0	+0.272	+0.287	—0.662	+3.647	3.360	......	......	.....	+0.328	—0.521	......
12...	0.25	3.2	—0.328	+0.521	—2.400	+4.671	4.150	......	......	.....	+0.134	+0.675	......
13...	0.25	3.2	−0.134	—0.675	—2.060	+3.475	4.150	......	......	.....	—0.263	—0.345	......
14...	0.25	3.2	+0.263	+0.345	—1.362	+4.495	4.150	+0.988	+0.445	0.443	+0.435	—0.232	44.66

CONCLUSIONS

In this paper the purpose is simply to present the method of analysis, and no effort has been made to draw definite conclusions regarding the practicability of the flexible-base type of building construction. Certain tentative conclusions may be drawn, however, from the preceding computations and from similar computations that have been made for other sets of conditions. These may be briefly summarized as follows:

1.—The period of acceleration is as important as the acceleration itself, in its effect on the deflection of the structure. A period of 0.7 sec instead of 1.0 sec, cuts the deflections roughly in half.

2.—The impact effect of an acceleration suddenly applied to a flexible structure of this type has little or no effect on the deflection.

3.—An irregular swing gives greater deflections than a uniform and rhythmic swing, and within the range of probable maximum periods of acceleration there is no evidence of resonance if the free period of vibration of the structure is kept above 2.0 or 2.5 sec.

4.—The maximum deflection produced in a structure by a given ground motion, is not much affected by considerable changes in the period of free vibration of the structure. For this reason, the best measure of the earthquake-resistant qualities of a given building of this type is simply the maximum horizontal deflection that it can sustain without exceeding allowable stresses in the columns.

5.—It is likely to be found difficult to provide sufficient lateral flexibility in this type of structure if the most unfavorable period and other conditions of acceleration are assumed. Long base columns will have to be provided, probably with some form of hinged base, in which case the steel that must be added in these columns and in the restraining beams to which they frame at their top, in order to provide a reasonable resistance to wind pressure, will be found to be a large and important item.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

INVESTIGATION OF WEB BUCKLING IN STEEL BEAMS

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H. J. GODFREY,[2] ESQ.

SYNOPSIS

The results of an investigation on the web failure of structural steel beams are reported in this paper. Tests were made on rolled sections as well as on sections made from plates by means of electric arc welding. The depth-thickness ratio of the web of the beams varied considerably; however, all the beams gave indication of initial failure due to shear. At the initial failure of the beam (its yield point), the computed maximum horizontal shearing stress in the web was found to correspond very well with the shearing yield-point stress of the material. The conclusion is drawn that the shearing stresses developed in the web, rather than buckling, determine the usefulness of a beam for depth-thickness ratios of 70 or less. The trend of the relationship indicated that buckling of the web might occur at a depth-thickness ratio of approximately 80.

INTRODUCTION

Since the degree of web buckling in beams and girders has long been a moot question, and since experimental data are scarce, an investigation of this subject was an urgent need. A theoretical study[3] was presented by Professor S. Timoshenko in 1933, but as no experimental results were offered in support of this theory, the question remained unsolved. In that paper Professor Timoshenko presents such a complete discussion of earlier experimental results that the inclusion of a bibliography in this paper is obviated.

For the purpose of securing reliable experimental information, a cooperative investigation was undertaken by the Bethlehem Steel Company

NOTE.—Discussion on this paper will be closed in May, 1934, *Proceedings*.

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[3] "Stability of Plate Girders Subjected to Bending," by Prof. S. Timoshenko, Preliminary Publication, International Assoc. for Bridge and Structural Eng., Paris, 1932, p. 129; abstracted in *Civil Engineering*, August, 1933, p. 456.

and the Fritz Engineering Laboratory of Lehigh University, Bethlehem, Pa. All the beams where designed and furnished by the Bethlehem Steel Company and tested at the Fritz Engineering Laboratory. Special acknowledgment is made to Jonathan Jones and C. H. Mercer, Members, Am. Soc. C. E., Chief Engineer, and Consulting Engineer, respectively, of the McClintic-Marshall Corporation, and to V. E. Ellstrom, Manager of Sales Engineering, Bethlehem Steel Company, for their valuable assistance in carrying out the investigation.

Since the usefulness of a beam is determined by the maximum load it can sustain without excessive deflection, the determination of its yield point becomes the most important factor in the testing. Emphasis was placed, therefore, upon securing the actual yield-point strength instead of the ultimate. The ultimate load has little significance beyond the fact that it is a measure of the toughness of the beam after it has lost its usefulness. In the study of the data the yield-point strength of the beam was used as the criterion for its load-carrying capacity.

Program

In order to obtain a suitable method of testing for use in the major investigation, a preliminary series of tests was run. This series included four Bethlehem B12–28 rolled sections, two of which had free ends; the remaining two had steel plates welded to the end sections to prevent failure by end twisting. A 1-ft sample section of each beam was furnished for the preparation of test coupons, on which the tensile and shearing properties of the material were determined.

Two groups of beams, designed to secure a definite failure in the web, constituted the major investigation. The first group consisted of five beams, three of which were made by welding steel plates together; the remaining two were rolled sections reinforced with cover-plates welded to the flanges. The second group consisted of five beams, all of which were welded sections. Since the design formulas make the depth-thickness ratio of the web the criterion for the working stresses, beams having high ratios were included in these tests. The highest $\frac{h}{t}$ ratio for rolled sections was about 55, and for welded sections about 70. (In this paper, h represents the clear distance between the flanges, and t, the thickness of the web.)

Preliminary Investigation

Information relating to the four beams tested in the preliminary investigation is presented in Table 1(a). It is noted that these beams had an $\frac{h}{t}$ ratio of about 40. They were tested in the 300 000-lb screw-power machine, the load being applied at the rate of 0.05 in. per min. The sizes of all beams were measured with micrometers and were found to differ slightly from those listed in handbooks. The properties based on the actual dimensions are given in Table 1(a).

TABLE 1.—Test Results, Investigation of Web Buckling

Date of test, 1932	Beam No.	Net web area, ht, in square inches	Web ratio, $\frac{h}{t}$	Moment of inertia, I, in inches[4]	Section modulus, S, in inches[3]	Reaction, V, in pounds	Stresses, in Pounds per Square Inch: $\frac{V}{ht}$	$\frac{Vay}{It}$	$f=\frac{M}{S}$	Shear in web specimens	Tension in web specimens	Tension in flange specimens	Reaction, in pounds
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(a) Preliminary Investigation on 12-Inch 28-Pound I-Beams (B 12-28)													
April 1........	1	3.06	40.2	205.5	34.2	55 000	17 950	18 550	29 000	23 650*	....	37 340* 58 810†	60 000†
April 5........	2	3.06	40.2	205.5	34.2	60 000	19 600	20 250	31 600	22 500* 50 750†	....	37 970* 59 260†	64 500†
May 27........	3	3.02	40.8	200.0	33.33	65 000	21 550	22 400	35 100	22 300* 50 200†	40 800* 59 370†	39 950* 58 150†	70 750†
July 1........	4	3.02	40.8	200.0	33.33	65 600	21 700	22 600	35 400	24 500* 50 600†	39 550* 57 800†	....	72 225†
(b) Major Investigation, Group *A*													
June 15........	*WB*- 1	3.47	56.5	1 938	226	77 500	22 300	20 400	14 700	22 000* 53 500†	43 250* 62 000†	....	109 250‖
May 20........	*WB*- 2	3.57	54.9	1 955	228	95 000	26 600	24 400	17 900	24 500* 51 300†	47 810* 60 900†	....	127 800‖
June 16........	*WB*- 3	4.36	58.9	2 419	254	115 000	26 300	24 700	21 700	26 470* 51 600†	49 600* 61 425†	....	139 000‖
June 30........	*WB*- 4	12.41	54.7	3 342‡ 12 850§	240‡ 832§	280 000	22 500	24 200‡ 21 130§	20 200	22 000* 53 400†	43 500* 61 600†	....	298 800‖
June 16........	*WB*- 5	8.15	52.2	1 560‡ 6 922§	141‡ 550§	210 000	25 800	25 700‡ 23 250§	21 150	29 200* 55 630†	51 300* 66 650†	....	225 000‖
(c) Major Investigation, Group *B*													
November 30...	*WB*- 6	4.40	70.0	2 866	278.5	78 800	17 900	16 800	14 100	17 450* 46 900†	33 080* 46 800†	....	96 300‖
December 1....	*WB*- 7	3.88	60.6	2 216	241.8	74 100	19 100	17 800	13 800	18 600* 45 850†	33 700* 48 400†	....	95 050‖
December 2....	*WB*- 8	4.10	59.7	2 326	249.2	71 550	17 500	16 250	12 930	19 920* 46 250†	29 680* 46 080†	....	99 750‖
December 1....	*WB*- 9	3.12	50.0	1 523	196.5	59 450	19 050	17 250	12 100	18 480* 44 800†	30 280* 46 980†	....	92 000‖
December 2....	*WB*-10	3.14	49.4	1 520	196.5	60 900	19 380	17 580	12 400	18 400* 46 250†	30 270* 47 150†	....	94 000‖

* Yield-point stress. † Ultimate. ‡ At section having no cover-plates. § At section having cover-plates. ‖ Maximum.

The beams were supported by a roller at one end and by a spherical bearing-block at the other end. The load was applied through a roller and a spherical bearing-block at the quarter-points of the span. At that side of the center line where a roller was used as a support, a spherical bearing-block was used for the application of the load, and *vice versa*, as shown in Fig. 1. Lateral deflections of the web were measured by dial gauges placed

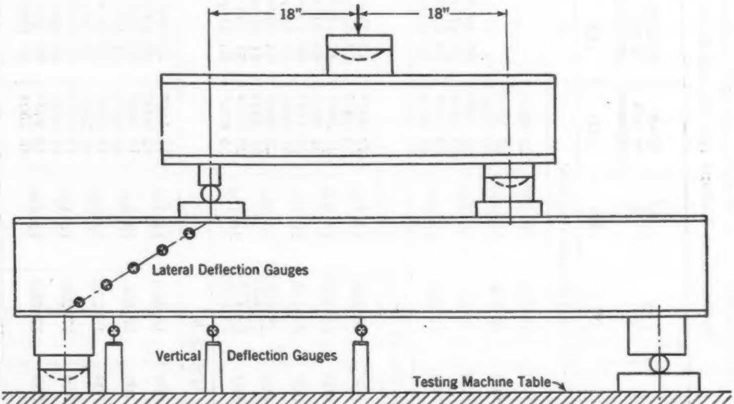

FIG. 1.—LOADING ARRANGEMENT FOR PRELIMINARY INVESTIGATION

on both sides of the web along a line connecting the loading point and the support. A movement of 0.0001 in. could be read directly by these gauges. On the first beam tested (No. 1, Table 1(*a*)), the gauges were supported by a frame attached to the table of the machine. This arrangement was not satisfactory, however, due to the relative movement between the beam and the table. For the remaining three beams the frame holding the gauges was clamped directly to the flanges of the beams. Vertical deflections were also observed on the two beams that had plates welded to the end sections.

In the first free-ended beam the center of the web was $\frac{3}{16}$-in. off center with respect to the top flange. The web of this beam began to scale at a load of 110 000 lb, which was taken as the yield point. However, it continued to take load until a total of 120 000 lb had been applied. At this load one end twisted sidewise. The eccentricity of the top flange may have contributed to an earlier twisting than would otherwise have occurred.

The greatest lateral web deflection occurred in Beam No. 2, Table 1(*a*), near the top flange where, at a load of 110 000 lb, the deflection was about 0.075 in. To a load of 60 000 lb the increase in lateral deflection was nearly constant for each increment of load. Beyond 60 000 lb the rate of increase in lateral deflection became greater for each additional increment. Strain-gauge measurements were also taken at points shown in Fig. 2, by means of a 2-in. strain-gauge. The compressive strains obtained

are shown in Fig 3. It is noted that at a load of 110 000 lb none of the gauge lines showed strain near the yield-point strain of the material in the web, but there is a tendency for the rate to increase. The beam first began to scale at a load of 120 000 lb which was taken as the yield point. The

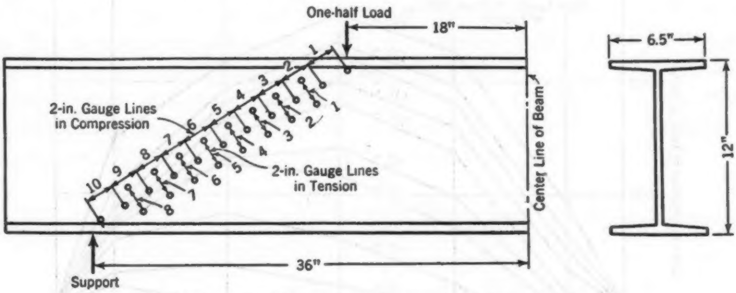

FIG. 2.—LOCATION OF STRAIN-GAUGE POINTS ALONG WEB

beam continued to take load until a maximum of 129 000 lb was reached, at which time the ends twisted sidewise.

Table 1(*a*) shows that the test coupons gave shearing yield-point stress in the web of 23 650 and 22 500 lb per sq in., respectively, for Beams Nos. 1 and 2. The average yield point and ultimate strength in tension, as de-

FIG. 3.—AVERAGE STRAIN IN COMPRESSION ALONG LINE CONNECTING LOADING POINT AND SUPPORT

termined by tests on the coupons from the flanges of these beams, were 37 700 and 59 000 lb per sq in. The shearing yield-point stress was thus about 64% of the yield-point stress in tension. The computed maximum shearing stresses in the web were 18 550 and 20 250 lb per sq in. at the

yield point of the two beams. The maximum fiber stresses in the flanges were 29 000 and 31 600 lb per sq in. at the yield point of the beams. These data show that the yield point of the material had not been reached, either

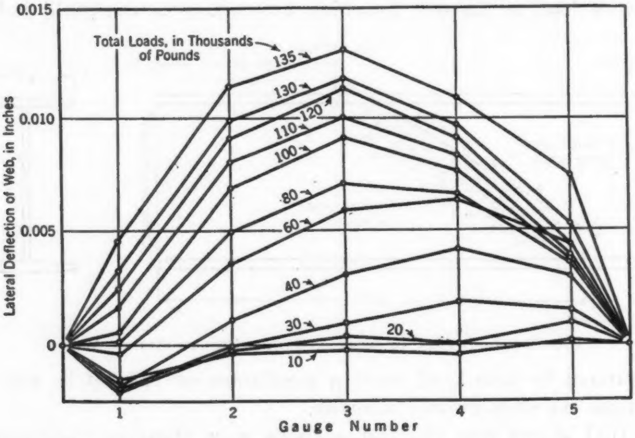

FIG. 4.—LATERAL DEFLECTION OF WEB OF BEAM NO. 3, TABLE 1

in shear or in tension, at the load at which scaling occurred. Furthermore, the failure was due to end twisting instead of web buckling or shear. In

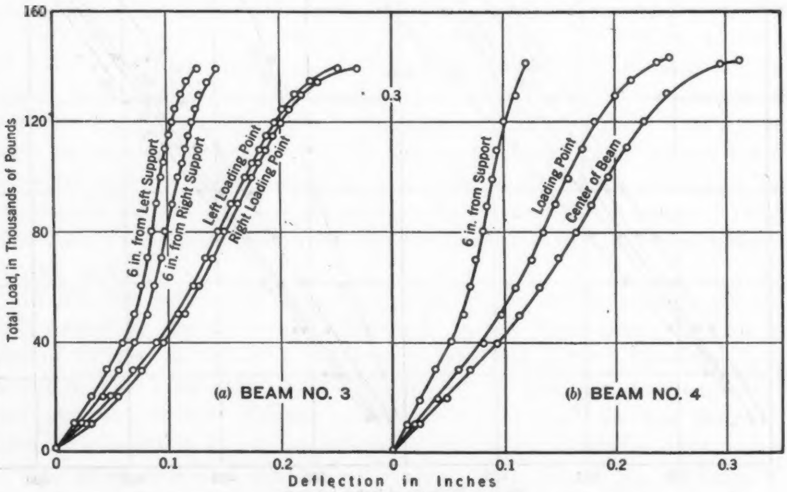

FIG. 5.—DEFLECTION CURVES FOR BEAMS NOS. 3 AND 4, TABLE 1 (ULTIMATE LOAD, BEAM NO. 3, 141 500 POUNDS; BEAM NO. 4, 144 450 POUNDS)

order to prevent end twisting steel plates were welded to the ends of the remaining beams.

Both vertical and lateral deflections were observed for the beams with end plates. These beams were whitewashed before the testing so that strain

lines could be observed more readily. The loading arrangement is shown in Fig. 1. Th lateral web deflections for Beam No. 3, shown in Fig. 4, were similar in shape to those for Beam No. 4. However, the lateral deflection of Beam No. 4 became less after a load of 50 000 lb had been applied, whereas in Beam No. 3 the deflection increased throughout the test. It should be noted that at a load of 135 000 lb, the lateral web deflection of Beam No. 3 was only 0.013 in. The similarity of the vertical deflection curves was also very marked, as shown in Fig. 5. The yield point for these two beams was taken as the point at which the slope of the tangent to the deflection curve was twice as large as the slope of the straight part of the curve. The loads at the yield points were 130 000 and 131 000 lb.

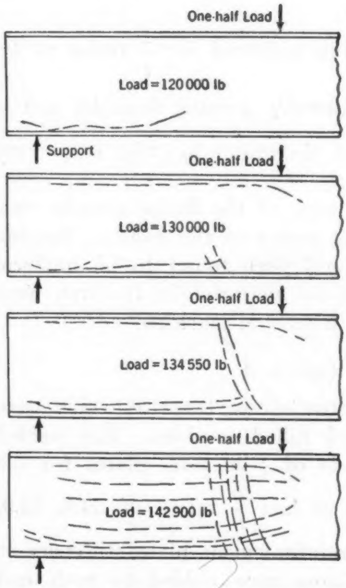

FIG. 6.—FLAKING OF WHITEWASH ON BEAM NO. 4

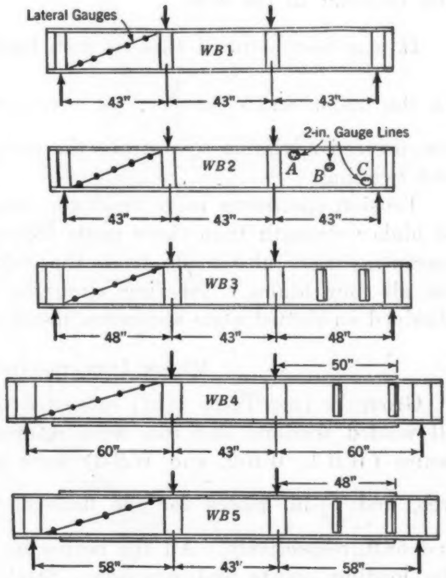

FIG. 7.—MAKE-UP AND LOADING ARRANGEMENT FOR MAJOR INVESTIGATION, GROUP A

The beams continued to take load beyond the yield point until maximum loads of 141 500 and 144 450 lb were reached, at which time, end twisting occurred in a manner similar to Beams Nos. 1 and 2.

At a load of about 110 000 lb, the first strain lines appeared on both beams in the form of horizontal lines, at the root of the web near the support. With increased load, more strain lines appeared over the support and also below the loading point. At loads of 138 500 and 134 500 lb, respectively, for Beams Nos. 3 and 4, the yielding was so great that it produced a drop in the lever of the testing machine. Further increase in the load produced vertical strain lines in the web between the support and the loading point. All these lines were evidently due to shear. An illustration of the progressive formation of the strain lines is shown in Fig. 6.

Computed maximum shearing stresses in the web at the yield point, were found to be 22 400 and 22 600 lb per sq in. for Beams Nos. 3 and 4. These agree closely with the yield-point stresses in shear (22 300 and 24 500 lb per sq in.) obtained on the test coupons. At the yield point of the beams the maximum fiber stresses in the flanges were 35 100 and 35 400 lb per sq in. Since the tensile yield-point stresses of the coupons were 40 800 and 39 550 lb per sq in., the yield point of the beam was not caused by the flexural stresses. If the flexural stresses were computed for loads corresponding to the drop of the beam, they would still be less than the yield-point stress of the material. It may be concluded, therefore, that the yield point of these beams was determined by the yield point in shear of the material in the web.

It was demonstrated that no web buckling appeared for $\frac{h}{t}$ ratios of 40. In the major series the $\frac{h}{t}$ ratios were considerably greater than 40, and it was deemed advisable to restrain the ends of the beams in order to prevent end twisting.

Tension specimens made from the outer edge of the flange usually were of higher strength than those made from the center of the flange. Tension specimens were also made from the web, and were found to be uniform for all four beams. Shearing strengths of the material in the web were obtained on slotted plate specimens tested in a tension machine.[4]

MAJOR INVESTIGATION, GROUP *A*

Group *A* (see Table 1(*b*)) consisted of five beams, three of which were all welded sections and two were reinforced rolled sections. The welded beams (*WB*-1, *WB*-2, and *WB*-3) were made of $\frac{1}{4}$-in. tank plates for the web, and $1\frac{1}{2}$-in. plates for the flanges; these had $\frac{h}{t}$ ratios of 56.6, 54.9, and 58.9, respectively. All the beams had reinforcement plate stiffeners at the loading points and supports. Steel plates were welded to each end section to prevent end twisting. In addition, Beam *WB*-3 had two angle stiffeners on each side of the web in one of the panels between the support and the loading point. The make-up and the loading arrangement for these beams are shown in Fig. 7. Both vertical deflections of the beam and lateral web deflections were observed during the testing of these beams. The location of the gauges was similar to that of the beams in the preliminary tests. The beam designated in Table 1(*b*) as *WB*-2 was not whitewashed, and strain-gauge observations were taken with a 2-in. gauge length at points of the web, as indicated in Fig. 7. All other beams in this group were whitewashed and had no strain-gauge observations. The properties of the beams based on their actual dimensions, and the results of the tests are given in Table 1(*b*).

[4] Description of these tests is given in a paper entitled "Shearing Properties and Poisson's Ratio of Structural and Alloy Steels," by Inge Lyse, M. Am. Soc. C. E., and H. J. Godfrey, *Proceedings*, Am. Soc. for Testing Materials, Part II, p. 274, 1933.

For Beam *WB*-1 the first local strain lines appeared under the loading point near the junction of the web and the top flange, at a load of only 50 000 lb. At a load of 125 000 lb horizontal and vertical strain lines appeared in the panel between the loading point and the support. The size and number of strain lines increased with an increase in load. The yield point at 155 000 lb was determined from the deflection curve in the same manner as that used in the preliminary investigation. The beam continued to take load until a maximum of 218 500 lb was attained and, at this point, the load fell off, accompanied by a gradual sagging of the beam.

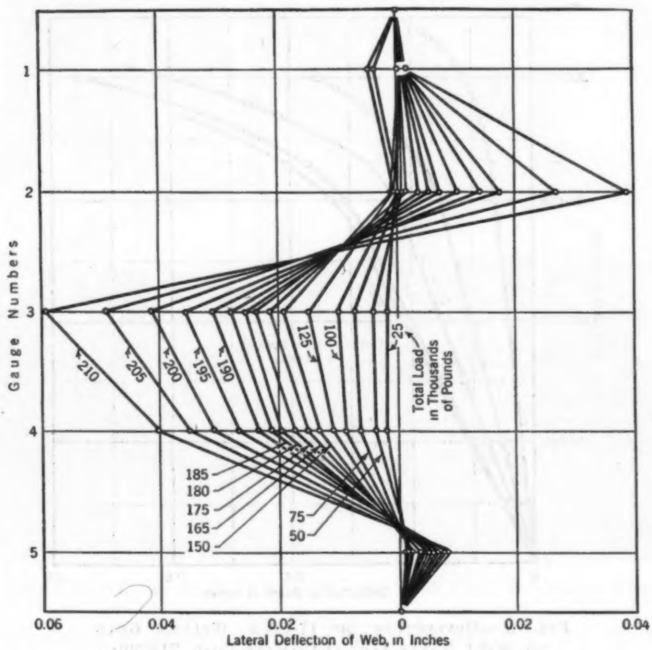

FIG. 8.—LATERAL DEFLECTION OF WEB OF 17-INCH WELDED BEAM NO. *WB*-1, TABLE 1(*b*) (ULTIMATE LOAD, 218 500 POUNDS)

Lateral web deflections for Beam *WB*-1 are given in Fig. 8. It is noted that the center of the web deflected in opposite directions to the deflections at the top and bottom. The web deflection at the yield point of the beam was only 0.02 in., indicating that buckling was not the cause of yielding. The vertical deflections of this beam are given in Fig. 9. It is seen that a fairly sharp increase in the rate of deflection took place at a load of 155 000 lb. This load produced a computed maximum shearing stress of 20 400 lb per sq in. The coupon gave a shearing yield-point stress of 22 000 lb per sq in., which does not differ greatly from the computed shearing stress at the yield point of the beam.

Fig. 10 shows the appearance of Beam *WB*-1 (Table 1(*b*)) at the yield-point load. It is to be noted that the horizontal and vertical strain lines are predominating. Furthermore, a number of local strain lines are grouped along the welds. After the beam had reached the maximum load, the buckling of the web could easily be seen.

Beam *WB*-2 was tested in a manner similar to Beam *WB*-1, except that strain-gauge observations were also taken. The location of the observation points is indicated in Fig. 7. The first strain lines on the web appeared at a load of 68 000 lb while the yield point, as determined from the deflection

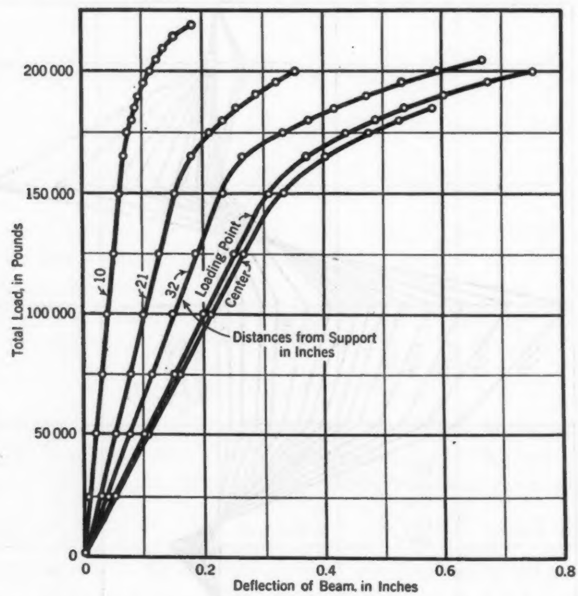

FIG. 9.—DEFLECTION OF 17-INCH WELDED BEAM No. *WB*-1, TABLE 1(*b*) (ULTIMATE LOAD, 218 500 POUNDS)

curve, was found to be at a load of 190 000 lb. The vertical deflections are shown in Fig. 11. The beam continued to take load beyond the yield point and reached a maximum of 255 600 lb, at which time a slight buckle could be seen in one of the web panels. The computed maximum shearing stress at the yield point of the beam was 24 400 lb per sq in. This compared very well with the shearing yield-point stress of 24 500 lb per sq in., obtained on the coupons. The strains obtained by means of the 2-in. gauge are plotted in Fig. 12. It is noted that certain gauge lines showed strains approaching the yield-point strains at a load of 175 000 lb.

Welded Beam *WB*-3 was tested in the same manner as Beam *WB*-1. The first local strain lines appeared at a load of 38 000 lb. After a load of 50 000 lb had been applied the load was released to 1 000 lb and a set

reading observed. This was also done after every following increment of loading up to the maximum load. The extent to which the lateral web deflections became set increased considerably after the yield point of the beam had been reached. The vertical deflections in Fig. 13, showed only very small sets at low loads. The yield point of the beam was reached at a load of 230 000 lb, and the vertical permanent set increased considerably after this load had been exceeded. The computed maximum shearing stress at the yield point of the beam was 24 700 lb per sq in. This compares

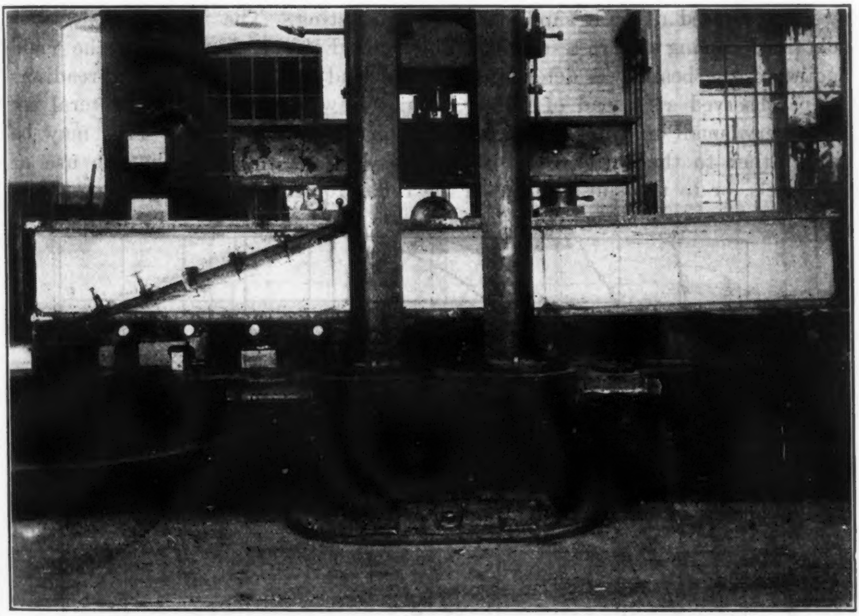

FIG. 10.—WELDED BEAM, *WB*-1, SHOWING WEB FAILURE DUE TO SHEAR

favorably with a shearing yield-point stress of 26 470 lb per sq in. for the coupons. While there is no indication of web buckling at the yield point of the beam, definite web buckling appeared at a maximum load of 278 000 lb. Welded beams proved very satisfactory and showed no indication of distress in the fillet welds.

Beam *WB*-4 was a rolled Bethlehem B28-91 section, the make-up of which is shown in Fig. 7. This beam had plate stiffeners welded to the web at points of support and loading and also had riveted angle stiffeners in one of the end panels. In order to prevent flexural failure, cover-plates that extended to within a short distance of the supports, were welded to the flanges.

Due to the size of this beam, it was necessary to use the 800 000-lb testing machine. As the load was applied the entire beam deflected sidewise to some extent until at a load of 475 000 lb that end of the beam

which was supported by the spherical bearing-block twisted. The deflections showed that up to the load at which twisting occurred there was no indication of yielding.

Since the twisting of one end of the beam evidently was due to the loading arrangement, it was deemed advisable to re-test this beam under more favorable conditions. Consequently, lateral restraint for the ends of the beam, in the form of channels bolted to the supporting beam, was used in the re-testing. The friction between the upper flange of the beam and the channels was kept at a minimum by the use of rollers, which were also used at both supports in the re-testing. The vertical deflections obtained during the re-testing are shown in Fig. 14, from which the yield point of the beam was determined at the load of 560 000 lb. Set readings were observed at a load of 25 000 lb, and it was noted that the lateral set was very small in comparison to the lateral web deflection. This may be due either to the better restraint on the ends of the beam by the use of channels, or to the substitution of the roller for the spherical bearing-block.

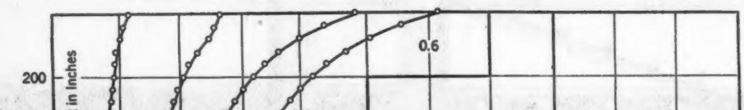

FIG. 11.—DEFLECTION OF 17-INCH WELDED BEAM NO. *WB*-2 (ULTIMATE LOAD, 255 600 POUNDS).

FIG. 12.—AVERAGE TENSION AND COMPRESSION STRAINS ON WEB OF BEAM NO. *WB*-2

The beam began to twist at a load of 325 000 lb. This rotation continued with the increase in load until at the maximum of 597 600 lb the beam twisted so much that the entire loading rig was out of position. At the maximum load, the web did not scale except on the stiffeners and along the junction between the web and the flange.

At the yield-point load the computed maximum shearing stress in the web was 24 200 lb per sq in. in the section between the support and the cover-plates on the flanges, and 21 130 lb per sq in. in the section having cover-plates. The yield-point stress in shear obtained from tests of coupons was 22 000 lb per sq in. For this beam also, the shearing yield-point stress obtained from the material agreed very well with the shearing stress computed for the web at the yield point of the beam.

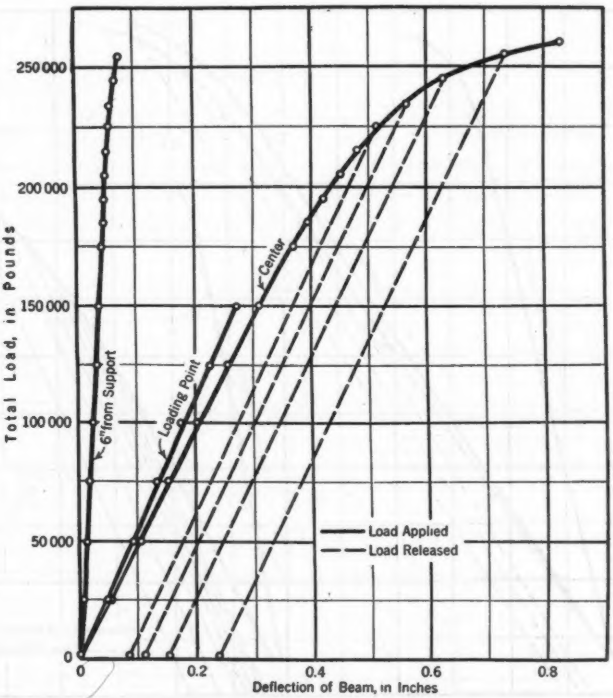

FIG. 13.—DEFLECTION OF 19-INCH WELDED BEAM NO. *WB*-3 (ULTIMATE LOAD, 278 000 POUNDS)

Beam *WB*-5 was a 22-in., 62-lb I-beam reinforced with plate stiffeners and cover-plates, as shown in Fig. 7. The first appearance of scaling on the web was discovered at a load of 50 000 lb. As the load was increased, the scaling increased, especially at the stiffeners. The strain lines on the web were primarily horizontal, indicating shearing stress. Since the scaling had no relation to the yield point of the beam, it may have been caused by high internal strains in the material. The vertical deflections and sets are given in Fig. 15, from which the yield point of the beam was determined at a load of 420 000 lb. The lateral deflections increased until a load of 275 000 lb was reached. For greater loads there was a tendency for the web to return to its original position. The beam continued to take

load until a maximum of 450 000 lb was reached, at which time it continued to deflect vertically without any increase in the load. It showed no indication of web buckling, even at the maximum load when most of the whitewash had flaked off in the parts between the support and the loading point.

At the section of the web between the support and the cover-plate, the computed maximum shearing stress at the yield point of the beam was

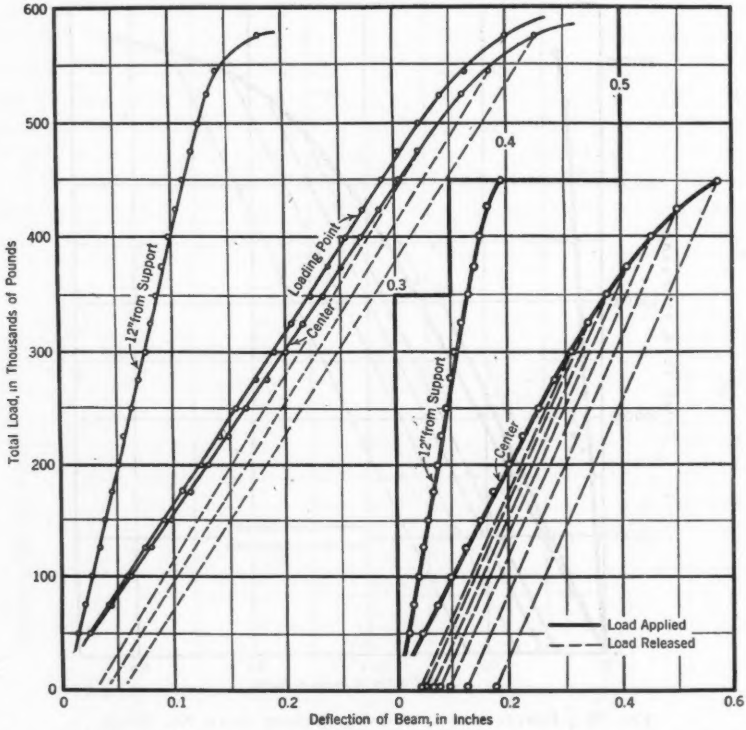

FIG. 14.—DEFLECTION OF A REINFORCED B-22-INCH, 62-POUND, BETHLEHEM I-BEAM, No. *WB*-5 (ULTIMATE LOAD, 450 000 POUNDS)

FIG. 15.—DEFLECTION OF A REINFORCED B-22-INCH, 62-POUND, BETHLEHEM I-BEAM No. *WB*-5 (ULTIMATE LOAD, 450 000 POUNDS)

25 700 lb per sq in. and 23 250 lb per sq in. in the section having cover-plates. These values are somewhat less than the shearing yield-point stress of 29 200 lb per sq in. for the coupons, which was considerably greater than any of the values obtained from coupons of the other beams. The latter showed shearing yield-point stresses between 22 000 and 26 470 lb per sq in. It seems, therefore, that the value of 29 200 lb per sq in, may be somewhat in error. For Beams *WB*-1 to *WB*-4 the ratio between the yield point in shear and that in tension varied between 0.506 and 0.534. However, the ratio obtained for Beam *WB*-5 was 0.569, which is considerably more than

the other ratios. Granting that the yield-point stress of the coupons was in error, Beam *WB*-5 also showed a fair agreement between the computed maximum shearing stress at the yield point of the beam and the shearing yield-point stress of the material.

As seen from Table 1(*b*), the computed maximum fiber stress in the flanges at the yield point of the beams in this group was only 21 700 lb per sq in. This is so much less than the yield point of the material, which had for a minimum 43 500 lb per sq in., that it is quite evident that the beams did not yield in flexure.

MAJOR INVESTIGATION, GROUP *B*

Group *B* consisted of five welded beams (*WB*-6 to *WB*-10, Table 1(*c*)), all of which were whitewashed before being tested in the 300 000-lb machine. The load was applied to the center of the beam through a spherical bearing-block, and rollers were used at both supports. To prevent end twisting, plates were welded to each end section of the beams, as was done in

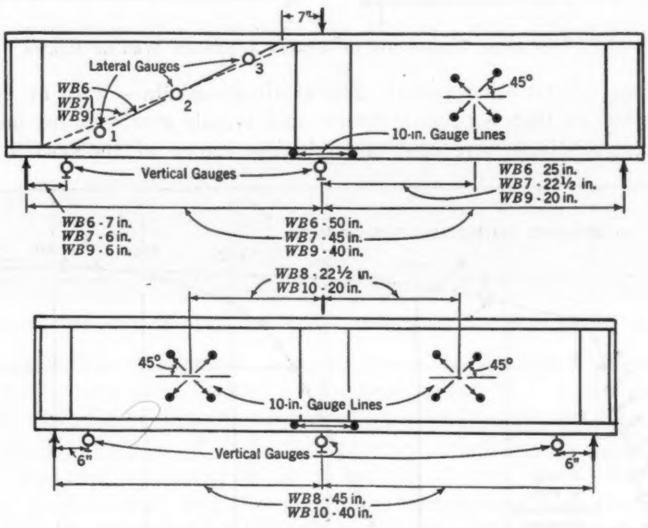

FIG. 16.—MAKE-UP AND LOADING ARRANGEMENT FOR MAJOR INVESTIGATION, GROUP *B* BEAMS

Group *A*. Plate stiffeners were welded on the web at the supports and loading points so that local failure would not occur. The make-up and loading arrangement of these beams are shown in Fig. 16. Vertical and lateral deflections were observed, and strain-gauge readings were taken on various parts of the beams by a 10-in. Whittemore strain-gauge.

Table 1(*c*) gives the properties of these beams based on actual dimensions. It is to be noted that the $\frac{h}{t}$ ratios ranged from 49.4 to 70.0. The mild steel plates used for the webs were found to be very ductile for all the

beams in this group. Consequently, the lever of the testing machine dropped decidedly at a load that was taken as the yield point of the beam.

Beam *WB*-6 had a web ratio of 70. The lateral gauges were placed along the web in one of the panels, and strain-gauge readings were taken

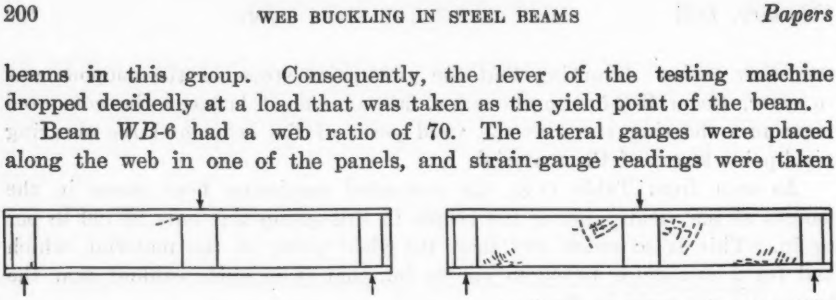

FIG. 17.—GENERAL FORMATION OF STRAIN LINES ON WEB OF BEAMS

in the center of the other panel. The strain-gauge lines were at 45° with the horizontal so that both compressive and tensile strains were measured. Strain-gauge readings were also taken in the center of the bottom flange,

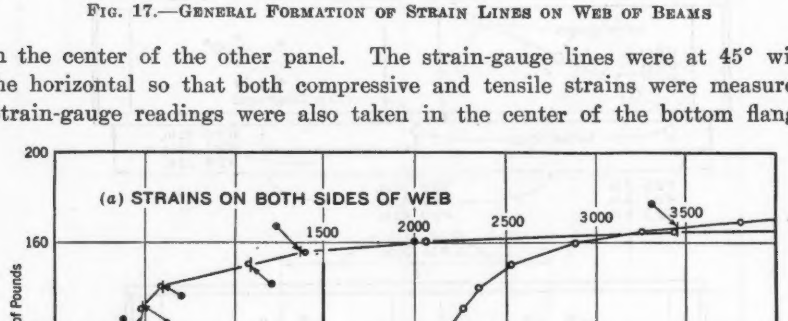

FIG. 18.—COMPRESSIVE STRAINS AND NET CENTER DEFLECTIONS, BEAM NO. *WB*-6 (ULTIMATE LOAD, 192 600 POUNDS)

in order to determine the maximum flexural stresses developed. The position of the gauges and strain-gauge holes are shown in Fig. 16.

Flaking occurred first at a load of 24 000 lb, in the form of vertical lines, at the top corner of the web below the loading point. As the load

increased, approximately vertical and horizontal strain lines extended across the web. At a load of 50 000 lb, strain lines appeared in the web near one of the supports. A gradual increase in the strain lines followed an increase in load. The general formation of strain lines as they appeared on the beams is shown in Fig. 17.

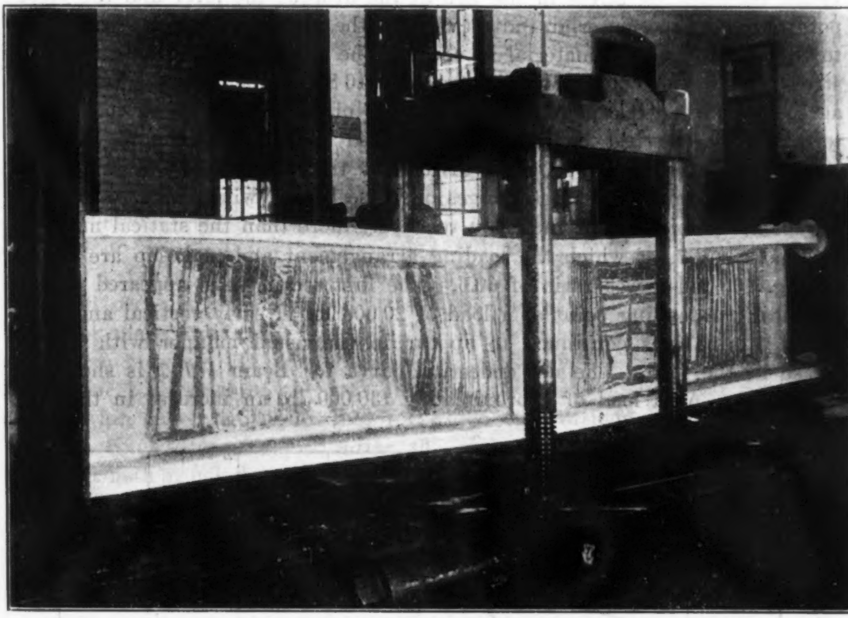

FIG. 19.—APPEARANCE OF BEAM NO. WB-6 AT THE MAXIMUM LOAD OF 192 600 POUNDS

Net deflections (difference between the center and end deflections) of Beam *WB*-6 are shown in Fig. 18(*b*), from which it is noted that an increase in the rate of deflection occurred at a load of 140 000 lb. At a load of 150 000 lb, the rate of deflection increased very sharply, and a decided drop of the lever was observed at a load of 157 600 lb. No buckling was observed at this load, and the maximum lateral web deflection was only about 0.008 in. at a load of 150 000 lb; it decreased with further increase in load. The strains observed for the flange indicated that the stress in the flange was very low at the yield point of the beam. The tensile strains in the web increased regularly until a load of 120 000 lb was reached. At 140 000 lb there was a decided increase in the tension strains. The compressive strains in the web are shown in Fig. 18(*a*), in which the strains on both sides of the web have been plotted separately in order to bring out the buckling behavior. It was found that the strains on each side coincide almost exactly throughout the test, indicating that no buckling took place within the loads for which observations were taken. Had buckling occurred, the strains on one side of the web would have increased much faster than those on the opposite side, due to the bending effect.

Fig. 18(*a*) is, therefore, a good illustration of the fact that no buckling took place at the yield point of the beam. The beam continued to take load until a maximum of 192 600 lb was attained. With further motion of the head of the testing machine the web in one of the panels buckled considerably, as shown in Fig. 19. Table 1(*c*) shows a close agreement between the shearing stress in the web at the drop of the lever and the yield-point stress of the material, indicating that the yielding of the beam was due to the yielding of the material in shear. The computed maximum shearing stress at the yield point of the beam was 16 800 lb per sq in. This value agrees with the yield-point stress in shear of 17 450 lb per sq in., as determined by the test coupons. It is noted that the maximum shearing stress is less than the total shear divided by the net area of the web. This is due to the unusually thick flanges on these beams, which tend to increase the moment of inertia of the beam relatively more than the statical moment.

Beam *WB*-7, of which the loading arrangement and make-up are shown in Fig. 16, had a web ratio of 60.6. The first strain lines appeared on the web near the loading point at a load of 20 000 lb. Nearly vertical and horizontal strain lines continued to appear in the usual manner with an increase in the load. The net deflection curve for Beam *WB*-7 is shown in Fig. 20(*b*). At a load of approximately 130 000 lb, an increase in the rate

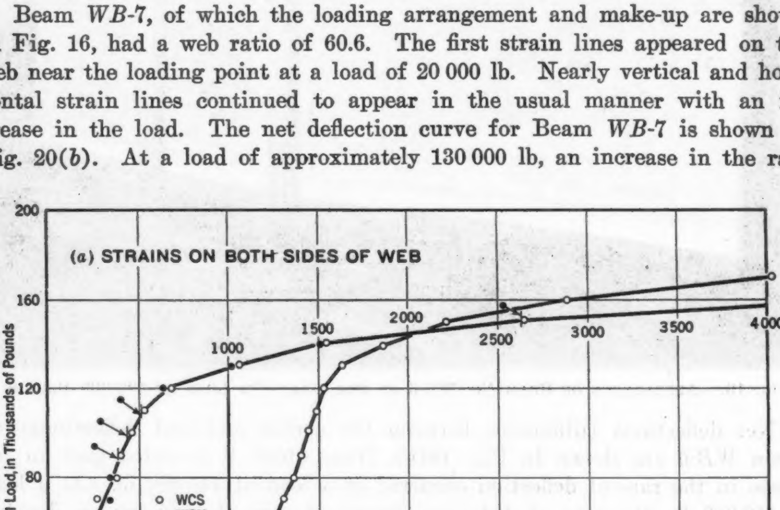

FIG. 20.—COMPRESSIVE STRAINS AND NET CENTER DEFLECTIONS, BEAM NO. *WB*-7 (ULTIMATE LOAD, 190 100 POUNDS).

of deflection was noted and, at a load of 148 200 lb, the lever dropped decidedly. The lateral deflection reached a maximum of about 0.025 in. at a load of 150 000 lb. A decided increase in the tension strains was noted at a load of 120 000 lb, while the flange strains showed no indication of yielding at the yield point of the beam. The compressive strains are shown

in Fig. 20(*a*). They were almost the same on both sides of the web throughout the test, indicating that no buckling occurred. A sharp increase in the compressive strains is noted at a load of 120 000 lb, which agrees with the behavior of the tensile strains. The beam continued to take load until one of the web panels began to buckle at a maximum load of 190 100 lb. The computed stresses are given in Table 1(*c*). The maximum shearing stress in the web at the drop of the lever was 17 800 lb per sq in., which agrees with the yield point in shear of 18 600 lb per sq in., as determined from the coupons.

Beam *WB*-8 was a companion of Beam *WB*-7 and had a web ratio of 59.7. Instead of lateral gauges being placed along one of the web panels, strain-gauge readings were taken in both panels to determine whether the load was evenly distributed on both sides of the loading point. The loading arrangement and make-up are shown in Fig. 16. This beam behaved similarly to Beam *WB*-7 throughout the loading, and the strain curves indicate a fairly even distribution of load. An increase in the rate of deflection occurred at a load of 130 000 lb, and a pronounced drop of the lever took place at a load of 143 100 lb. The tensile strains for both panels showed an increase in strain at a load of 120 000 lb. The compressive strains for both sides of the web also indicate that buckling did not take place within the range of loading for which observations were made. The beam continued to take load after the yield point was reached, until a buckle in both panels began at a maximum load of 199 500 lb. The results, as given in Table 1(*c*), show maximum shearing stress in the web of 16 250 lb per sq in. at the drop of the lever. This value is somewhat less than that obtained for Beam *WB*-7 and also less than the yield-point stress in shear as found by the test coupons.

Beam *WB*-9, the loading arrangement and make-up of which are shown in Fig. 16, had a web ratio of 50. Strain lines were visible on the web near the loading point at a load of 20 000 lb. Both vertical and horizontal strain lines appeared with an increase in load in the same manner as described for the other beams in this group. The deflection curves showed an increase in the rate of deflection at a load of 100 000 lb, and a decided drop of the lever took place at a load of 118 900 lb. A maximum lateral web deflection of 0.0045 in. was observed at a load of 130 000 lb, which indicates that buckling had not occurred up to that point. A decided increase in the web strains occurred at a load of 90 000 lb. The compression strains for both sides of the web indicate that at loads greater than the yield point of the beam they are slightly greater on the north side than on the south side. The maximum load carried was 184 000 lb, at which both web panels buckled. The results which are given in Table 1(*c*), show a good agreement between the shearing stresses in the web at the drop of the lever and the yield-point stress of the material. The maximum shearing stress was 17 250 lb per sq in., and the plate coupons gave 18 480 lb per sq in.

Beam *WB*-10 (Table 1(*c*)) was a companion to Beam *WB*-9. The first strain lines appeared on the web at a load of 20 000 lb near the loading

point. Vertical and horizontal strain lines appeared in the usual manner as the load was increased. The deflection curve showed an increase in the rate of deflection at a load of about 120 000 lb, and a drop of the lever occurred at a load of 121 800 lb. Both tensile and compressive strains in the web showed an increase in the rate of strain at a load of 100 000 lb. The agreement of the strains on both sides of the web indicated that buckling did not occur. The beam continued to take load until a maximum of 188 000 lb had been reached. The recults, which are given in Table 1(*c*), show a good agreement between this beam and its companion, Beam *WB*-9. The computed shearing stresses at the drop of the lever check the yield-point stress in shear as found by the test coupons.

In all these beams the maximum stresses in the flanges at the drop of the lever were less than one-half the yield-point stress in tension of the material.

EFFECT OF $h:t$ RATIO

In order to study the relation between the $\frac{h}{t}$ ratio and the shearing stresses developed, Fig. 21 was prepared. The beams in Group *B* were the only ones that lent themselves to such a study. It is noted that the shear-

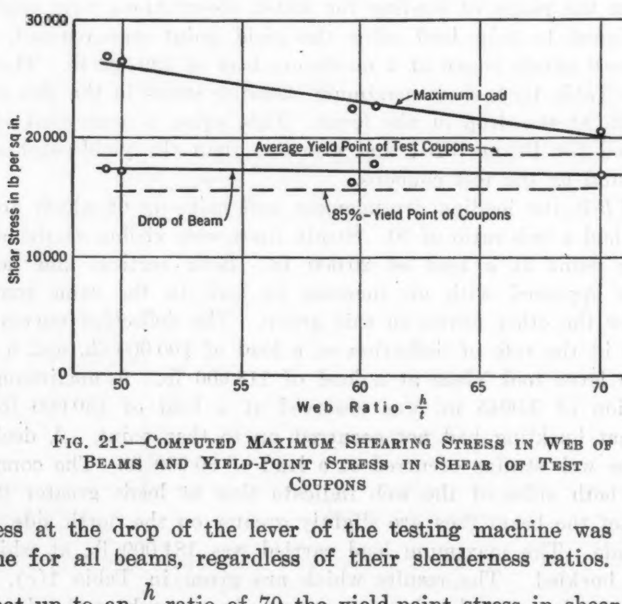

FIG. 21.—COMPUTED MAXIMUM SHEARING STRESS IN WEB OF BEAMS AND YIELD-POINT STRESS IN SHEAR OF TEST COUPONS

ing stress at the drop of the lever of the testing machine was very nearly the same for all beams, regardless of their slenderness ratios. This indicates that up to an $\frac{h}{t}$ ratio of 70 the yield-point stress in shear of the web material determines the useful load-carrying capacity of the beam. For $\frac{h}{t}$ ratios of 70, or less, there seems to be no reason for designing beams on the basis of web buckling.

The shearing stress at maximum load is also shown in Fig. 21. The stress developed in the web is greater for beams having a low slenderness ratio than for beams having a high ratio. The maximum shearing stress decreased quite regularly with the increase in the $\frac{h}{t}$ ratios, indicating that the toughness of the beam decreased with the increase in the ratio.

If the web had buckled in any of the beams, the maximum load would have been equal to, or less than, the yield-point load. The trend of the relationship indicates that buckling of the web might occur at an $\frac{h}{t}$ ratio of about 80. This value compares very favorably with Professor Timoshenko's theoretical value of 90 for an assumed yield-point stress in shear of 20 000 lb per sq in. for structural steel.

Summary

Although the number of beams tested in this investigation was too small on which to base final conclusions, the results obtained indicated that:

1.—The beams with free ends did not develop the full yield-point strength of the material, either in shear or in tension, due to failure in end twisting.

2.—At the yield point of all the beams which had plates welded to the ends, the computed maximum shearing stress in the web corresponded very well with the yield point in shear of the material.

3.—No beam showed any evidence of buckling at, or below, its yield point, indicating that with $\frac{h}{t}$ ratios up to 70 there is no danger of web buckling. Buckling may be expected to occur at $\frac{h}{t}$ ratios of 80 or more.

4.—The yield point of the beam, rather than the maximum load, should be used as a criterion for the factor of safety. In general, the average shearing stress in the web should be based on net area; that is, on $\frac{h}{t}$, rather than on gross area, $D\ t$.

5.—The first appearance of strain lines had no relation to the yield point of the beam.

6.—The yield point of the beam was not affected by the $\frac{h}{t}$ ratio of the web. The maximum load, however, decreased with an increase in the $\frac{h}{t}$ ratio.

7.—The shearing yield-point stress of the material was the important factor for all the beams included in this investigation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

FOUNDATION TREATMENT AT RODRIGUEZ DAM

Discussion

By Charles P. Williams, M. Am. Soc. C. E.

Charles P. Williams,[7] M. Am. Soc. C. E. (by letter)[7a].—In view of the somewhat radical departure from conventional types of foundations for buttressed dams, the writer would have appreciated a more critical discussion by those who heard, or have read, the paper and have viewed the work during construction. Nevertheless, he appreciates the comments of those who entered into the discussion.

The building of any storage dam is a serious responsibility, especially in a region subject to earthquakes and at a site traversed by a geologic fault. The construction at such a site necessarily involves some risk. However, the region around the Rodriguez Dam is dependent absolutely upon storage for its water supply. Ground storage sufficient to meet the requirement cannot be developed. Periods of five years in which there is no stream flow are of common occurrence. Records show one period of seven years during which the run-off was insignificant. The construction of reservoirs at the best, or at the least objectionable, sites is a necessity. The question is whether the risk is too great to justify continued development and, if not, what type of structure shall be built in order to secure the greatest practicable insurance against damage.

No comment on the discussions submitted appears to be necessary. At the time of the presentation of the paper at the meeting of the Irrigation Division, in Sacramento, Calif., in April, 1930, some comment was presented relative to the geology of the site. It was suggested that the rift in the bed-rock at the Rodriguez site is not a true fault, but a tensile crack, the main fault lying between the mainland and the Coronado Islands; that the type of dam selected is not suited to the site; and that a more flexible type, such as an earth or rock-fill dam, should have been chosen.

Note.—The paper by Charles P. Williams, M. Am. Soc. C. E., was presented at the meeting of the Irrigation Division, Sacramento, Calif., on April 24, 1930, and published in October, 1932, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1933, by Messrs. L. A. Robb, Francisco Gomez-Perez, and F. A. Noetzli.

[7] Cons. Engr., Los Angeles, Calif.

[7a] Received by the Secretary January 12, 1934.

The fact that the rift at the dam site is a true fault, and not a mere tensile crack, is proved by the occurrence of dense clay gouge, as much as 1 ft thick, along the faces of the hanging wall and foot wall.

At the time of the preliminary examination, the writer suggested an earth and rock-fill dam as being the type probably best suited to the site. This recommendation was made without knowledge of the existence of a fault, but with the knowledge that borings indicated, for a part of the foundation area in the stream bed, underlying disintegrated material extending to a depth of more than 100 ft. The writer had no part in the selection of the type of dam which eventually was chosen; however, in the light of subsequent determination of the character of the foundation site, he is of the opinion that the selection was fortunate. Excavation disclosed the fault, not discovered in the preceding geological examination. The question of shearing movement along the fault then became one of paramount consideration.

The two geologists who examined the site, after the discovery of the fault, agreed that subsequent movement along the fault is not probable, although it is a possibility. If, then, a dam is to be built at such a site, the question arises as to the type of structure which will restrict the damage to the least possible amount should geologic movement occur. Such a movement might be either a vertical or a horizontal slip, or it might be a separation of the fault walls, forming a crack.

The effect of the earthquake of 1906 on the San Andreas and Upper Crystal Springs earth dams, near San Francisco, Calif., is sometimes cited as evidence of the relative safety of earth dams from damage by seismic disturbance.[8] At the time of the 1906 earthquake, the San Andreas Dam, which then had a maximum height of 95 ft above the stream bed, consisted actually of two dams, separated by a hard narrow ridge through which passes the San Andreas fault. A horizontal slip of about 8 ft occurred at the dam, but this slip was along the fault and did not pass through either of the dams. The structures were somewhat cracked by the earthquake, but failure did not occur although the reservoir was full at the time. The results would have been very different, however, had the slip passed through the dam proper. The Upper Crystal Springs Dam, having a maximum height of 85 ft above the stream bed, is crossed by the San Andreas fault. At the time of the 1906 earthquake, there was a horizontal slip of 8 ft through the body of the dam. This dam lies between the Upper and the Lower Crystal Springs Reservoirs and, at the time of the earthquake, the water stood at the same elevation above and below the dam. Very different results might have been expected, if the dam had been subject to unbalanced hydrostatic pressure.

It is inconceivable that a slip, such as occurred at the San Andreas and Upper Crystal Springs Dams, could occur in an earth dam of the height of the Rodriguez Dam, with a large quantity of water in storage, without failure resulting from the admission of water into the body of the dam through cracks, or, possibly, in the case of a vertical slip, by overtopping. After initial failure, progressive failure would be rapid, resulting in early collapse.

[8] *Engineering News-Record*, August 25, 1932, p. 218, and September 29, 1932, p. 385.

The objection to an earth dam, at this site, applies, in a somewhat less degree, to a rock-fill dam.

The Ambursen type of dam is more flexible than any other type of concrete dam, and small relative movements can occur without resulting in serious torsional stresses. If, however, a slip along the fault should occur, there is possibility of local failure. The failure of one or two buttresses might occur, without affecting seriously the remainder of the structure, except by progressive undermining, which probably would be slow. The dam might be even torn apart, but the separated sections, not being dependent for stability on the part destroyed, would still stand and would retard to a considerable extent the escaping water.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

STUDY OF STILLING-BASIN DESIGN

Discussion

BY C. MAXWELL STANLEY, JUN. AM. SOC. C. E.

C. MAXWELL STANLEY,[11] JUN. AM. SOC. C. E. (by letter)[11a].—Discussions of this subject served to emphasize certain parts of the paper and to raise certain questions regarding the applicability of the methods of design suggested therein. The writer wishes to express his appreciation of these contributions.

The discussion, by Mr. Lane, of the test data from the Cle Elum Dam of the U. S. Bureau of Reclamation is particularly interesting. The results of these experiments in stilling-basin design check closely the results submitted in the paper. Mr. Lane points out that in the model study of the stilling basin for the Cle Elum Dam, the results parallel the inclined position of the solid line on Fig. 7, which divides the satisfactory from the unsatisfactory cases, but that lower values of the ratio, $\frac{L}{DW}$, were obtained. He suggests three factors which may account for this difference: (1) The flatter incoming slope which would allow a more efficient hydraulic jump; (2) the fact that the jump would begin up stream from the point at which the pool was assumed to begin; and (3) the effect of the Cle Elum type of pool may continue beyond the end of the floor as the sill does not obstruct the flow but only protects the bed of the river from the scour.

There is little question but that the flatter incoming slope will allow the formation of a more efficient jump. This fact was encountered in the writer's experiments and in the analysis of the effect of the shape of the dam he indicated the possibility that a flatter slope would allow the jump to form more easily, thus decreasing the length of basin needed for the complete formation of the jump. The fact that the jump will form on a slope also produces the effect of requiring a shorter stilling basin. The third condition (the possibility of action continuing below the end of the pool), is related to the effect of submergence upon the weir. As mentioned in the paper, experiments were run with the weir submerged and while these experiments were

NOTE.—The paper by C. Maxwell Stanley, Jun. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April 1933, by Messrs. E. W. Lane, C. C. Inglis, and F. Knapp.

[11] Engr. (Young & Stanley, Inc.), Muscatine, Iowa.

[11a] Received by the Secretary January 15, 1934.

not sufficiently extensive to permit any complete analysis, it was very apparent that with a depth of water below the weir sufficient to submerge it, the capacity of a given stilling basin to dissipate energy was increased.

The writer would like to suggest a fourth reason why the results of the Cle Elum tests indicated satisfactory operation at lengths somewhat less than those found in his own experiments. In the latter experiments satisfactory operation was separated from unsataisfactory operation merely by observation of the flow over the weir. If this flow was smooth, indicating that turbulence had been removed, the action was classified as satisfactory. It was hoped to supplement this with further experiments using an erodible bed below the weir. It is the writer's opinion that it is unnecessary to remove all the turbulence from the water prior to passing over the weir in order to avoid erosion below it. This should be particularly true with submerged conditions similar to those used on the Cle Elum test.

The effect of the weir is twofold: First, it creates a certain pool depth, and, second, it acts as a sill to create an eddy, or roller, about a horizontal axis immediately down stream from the weir. The direction of this roller is such that the water in contact with the bed of the river flows up stream toward the sill and thus prevents erosion. This phenomenon has been observed in numerous model studies and advantage has been taken of it in the design of many protective devices below spillways.

Mr. Inglis questions the applicability of the data to any except a few special cases. He brings out the fact that dissipation of energy is a three-dimensional problem. This is true, but if the spillway, or stilling basin, is of any considerable length, the eddies or currents at the side will affect only a limited length of the basin and the major part of the design must be based on the two-dimensional condition similar to that studied. Proper care must be taken of the action of the side currents and eddies. Such actions will be special cases for every project as they are concerned with local topography to a considerable extent.

Mr. Knapp does not feel satisfied with the distinction between Types 1 and 2 and suggests that Type 1 can be considered as jump action with the nappe submerged by tail-water, while Type 2 represents free nappe with repelled jump. The difference in action between Types 1 and 2, as shown in Figs. 1 and 2, is very distinct, and there was nothing in the appearance of the action to suggest that Type 1 is a form of jump action. It is merely a stream of water shooting into a pool, flowing along the bottom of the pool, and, at the same time, creating a roller or eddy about a horizontal axis.

One point, which has been stressed by Mr. Lane and also by the writer, is worthy of further mention. This is the fact that the design of a stilling basin for any particular project is not only a matter of hydraulics, but also one of economics. For any condition there will be a number of combinations of length, depth, and submergence which will dissipate satisfactorily the energy from the falling water and the choice of a particular design will depend on the relative costs. Hence, it is an economic problem.

In conclusion, the writer wishes to emphasize again the need of additional research to determine more clearly a rational method of design of stilling basins. The need of research on the effect of different slopes of the dam and upon the submergence of the basin is particularly apparent from the tests on the Cle Elum Dam.

If this paper serves to stimulate further research in this field, it will indeed have served its purpose.

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DISCUSSIONS

STABILITY OF STRAIGHT CONCRETE GRAVITY DAMS

Discussion

BY MESSRS. PAUL BAUMANN, THADDEUS MERRIMAN, IVAN E. HOUK, A. V. KARPOV, L. F. HARZA, AND EDWARD GODFREY

PAUL BAUMANN,[36] M. AM. SOC. C. E. (by letter)[36a].—The crushing test, simple though it seems on the face of it, is in fact very intricate. It is far from being satisfactorily interpreted by means of Coulomb's internal friction or wedge theory, which dates back to the third quarter of the Eighteenth Century and which is closely related to the wedge theory of earth pressure and earth resistance, respectively.

A somewhat later interpretation of the crushing test has been based on the postulate that the failure occurs along surfaces of maximum shear (principal shear), which leads to the contention that the shearing strength is one-half the first principal or direct stress in a mono-axial system. Both theories are approximations, the latter being fairly close for cubic specimens, but both gain in fallacy with a decrease in the restraint of the faces in contact with the metal shoes of the testing machine.

Due to the friction between concrete and steel and due to the much greater rigidity of the latter, the concrete is restrained from transverse expansion (Poisson's ratio) which causes a curvature of the sides and a stress condition which is no longer mono-axial.

Prior to failure, the specimen thus takes a shape as shown exaggerated in Fig. 10, and failure starts half way between the shoes where the effect of end restraint and the density of the material are a minimum. The shape of the surface of failure is the reflected shape, so to speak, of the free surface of the specimen. Tests conducted by Foeppl[37] showed that, with the contact faces of the specimen well lubricated, failure occurs along planes parallel to

NOTE.—The paper by D. C. Henny, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by H. de B. Parsons, M. Am. Soc. C. E., December, 1933, by Messrs. A. A. Eremin and Calvin V. Davis; and January, 1934, by Messrs. William P. Creager, F. W. Hanna, Lars R. Jorgensen, and I. M. Nelidov.

[36] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs, Consolidated, Los Angeles, Calif.

[36a] Received by the Secretary December 12, 1933.

[37] "Verlesungen über Technische Mechanik," Vol. 3, Tenth Edition, pp. 87-88.

the main (vertical) axis and divides the specimen into vertical prisms. The crushing strength in some cases was reduced to one-fourth that for non-lubricated contact faces. It would be rather difficult to explain this phenomenon by means of Coulomb's wedge theory.

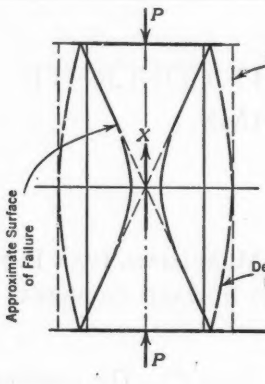

FIG. 10.

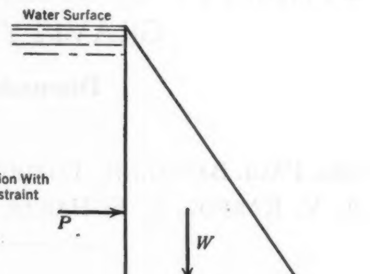

FIG. 11.

Similar results were obtained a short time ago with concrete cylinders of various sizes by J. Y. Jewett, Assoc. M. Am. Soc. C. E., in the testing laboratory of the City of San Diego, Calif.

A tentative study by Foeppl[38] on the crushing test without lubrication leads to the conclusion that the contact faces are only partly restrained and that the distribution of normal and tangential stresses there (Fig. 10) may be expressed as:

$$\sigma^0_x = -\frac{P}{4\,a^2} + 0.72\,C\left\{\cos\pi\frac{y}{a} + \cos\pi\frac{z}{a}\right\} \quad \ldots\ldots (24)$$

and,

$$\tau_0 = C\sqrt{\sin^2\pi\frac{y}{a} + \sin^2\pi\frac{z}{a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which, C is a constant that depends on the coefficient of friction between steel and concrete and which follows from the conditions, $\dfrac{\partial\left(\dfrac{\tau_0}{\sigma^0_x}\right)}{\partial y} = 0$ and $\dfrac{\partial\,\dfrac{\tau_0}{\sigma^0_z}}{\partial z}$

[38] "Drang und Zwang," Vol. 1, Second Edition, p. 116.

= 0; and that the maximum value of $\frac{\tau_0}{\sigma^0_x} = \mu =$ coefficient of friction between the steel and the concrete.

The author very ably treats the uplift problem from a standpoint of inner stability rather than overturning. As long as the unit weight of the dam is greater than that of the liquid it retains, and as long as the uplift pressure is governed by the water surface in the reservoir and the dam cannot slide without shearing, the influence of uplift is on the inner stability only, because no change due to uplift of the forces above the base of the section under consideration has taken place. If the resultant uplift force were replaced by the pressure of a jack (Fig. 11), the outer stability would certainly not be changed as far as overturning is concerned, but the distribution of normal and tangential stresses along the section would have changed due to a rotation of the section and a diminution of contact, respectively.

Levy's rule to make the compression in the concrete at the up-stream face at all elevations at least equal to the water pressure under maximum head has been taught in leading engineering schools of Europe as the rule and has been the basic design principle of nearly all the straight or quasi-straight gravity dams built since it was first advanced. Although this may be too severe a practice, it certainly helped to minimize failures and it should always be observed in locations subject to seismic disturbances.

The stress function of a gravity dam of triangular shape and infinite height and length is known, and through it normal and tangential stresses may be determined at any point; these are the criteria of structural safety. For a finite height (which, of course, is the case in practice), the influence of yielding foundations must be superimposed. This influence also affects the uplift studies. It is particularly noticeable at the heel and toe and relatively small changes there may greatly reduce the stresses which, for a right angle between up-stream face and rock surface, tend to assume alarming magnitudes. These studies are best made by means of photo-elastic tests on models.

With intelligent construction, the sliding factor is of less importance than the shear safety factor, except at the foundation in the case of a slope in a down-stream direction. The occurrence of resistance due to shear is then doubtful in most cases, while the existence of resistance due to a coefficient of friction of, say, 0.70 cannot be doubted, barring abnormal uplift conditions.

The author's simple and fundamental manner of treating the subject deserves acknowledgment.

THADDEUS MERRIMAN,[39] M. AM. SOC. C. E. (by letter)[39a].—The presentation of uplift, based on the experiments of Mr. Woodard, is an especially interesting part of this paper. Undoubtedly, pore-space pressures are present in all dams and act in an upward direction with magnitudes of the order stated.

[39] Cons. Engr., New York, N. Y.

[39a] Received by the Secretary, December 3, 1933.

That being the case, these pressures must also act horizontally and the load, *P*, in Fig. 5 may not be considered as applied on the water face of the dam. In other words, to whatever extent the pores of a dam are filled with water under pressure to that extent will the total resultant of the water loading be applied in a diagonally upward direction within the body of the structure.

It is not easy to follow the author's logic with respect to the subject of shear because it is more than probable that every failure of a brittle material is actually a failure in tension. As yet the science of materials has not been advanced sufficiently to enable one to think with certainty in terms of shear which, to this time, has eluded definition. The author, however, assumes the correctness of Coulomb's internal friction theory and combines it with a proposed "basic formula" which is simply a ratio of convenience between two total values, the one a shearing strength, the other a water loading. From the vantage point of a new formula thus constructed, he then shows in Table 9 that the "shear safety factor" for the Elephant Butte Dam is 7.3, whereas the "apparent factor of safety" is 1.27. Which of these values, if either, is correct and which of them best represents the actual stability of this structure?

Much of the present-day confusion in regard to the analysis of dams is attributable to the use of a concept which seeks to express a purely physical situation in terms of an abstract view of relative safety. The true stability of a dam is not to be stated as simply as by any "factor of safety."

IVAN E. HOUK,[40] M. AM. SOC. C. E. (by letter)[40a].—An excellent treatment of straight, concrete, gravity dam design is contained in this paper. The methods proposed for considering foundation uplift, pore pressure, shear resistance, and the total factor of safety of such structures against failure by sliding undoubtedly will find favor among those engaged on dam design and construction.

Engineers have long recognized that the principal factor of safety of a straight concrete dam against failure by sliding at the base lies in the shear resistance of the concrete and foundation materials rather than in the amount by which the coefficient of friction exceeds the calculated sliding factor. Shear resistance has always been important at the base of a dam, because of the fact that the concrete is poured on irregularly excavated foundation surfaces. During recent years shear resistance has also become important at horizontal construction joints above the base, due to the increased care in preparing joint surfaces during construction, the provision of adequate horizontal keyways at joint elevations, and the use of upward sloping joint surfaces near the down-stream face of the dam.

Undoubtedly, the science of dam design and construction has reached a stage where shear resistance should be considered more carefully in the case of important gravity structures. Because of relatively large available strength in shear it may be possible in certain cases to allow a slightly higher sliding factor than would otherwise be permissible. The writer is whole-heartedly

[40] Senior Engr., U. S. Bureau of Reclamation, Denver, Colo.

[40a] Received by the Secretary December 28, 1933.

in accord with the proposal to calculate shear-friction factors of safety in designing gravity dams. However, he does not believe that sliding factor calculations should be dispensed with entirely, even in important and costly structures where the designing engineer can feel reasonably sure of securing satisfactory construction operations, rigid inspection, and strict adherence to specified design details. Sliding factors should be computed in the future as they have been in the past; and the cross-sections of all straight gravity dams should be proportioned so that the calculated sliding factors will not exceed the coefficient of friction, even if relatively high shear resistances may be available in all parts of the structure.

There is something wonderfully reassuring about a low sliding factor. Only three factors are involved in its calculation: Water pressure, weight, and uplift. Of these, uplift is the only one that is not susceptible of exact determination. Consequently, if the foundation rock is satisfactory, the designer, by including a conservative allowance for uplift in his calculations, and proportioning the cross-section so that the sliding factor does not exceed the coefficient of friction, can be confident that the dam will not fail by sliding, regardless of any accident, oversight, or negligence in construction operations.

This may seem ultra-conservative. Nevertheless, the writer believes in being ultra-conservative in designing structures the failure of which may mean the loss of many human lives as well as much valuable property. The failure of the St. Francis Dam, in Southern California, a few years ago, although primarily due to poor foundation materials rather than to a lack of shear resistance or a high sliding factor, certainly illustrated the advisability of being ultra-conservative in high dam design. Moreover, the writer knows of a few comparatively high dams, built during recent years, in which, the horizontal construction joints are not capable of developing a high shear resistance. They are safe because they were designed with relatively low sliding factors.

A. V. KARPOV,[41] M. AM. SOC. C. E. (by letter)[41a].—An interesting and timely symposium on modern data and ideas as to the shearing strength of concrete and the influence of uplift force on stability of dams is presented in this paper. At the same time the paper is a valuable expression of the dissatisfaction that has been felt for a long time with the present methods of dam design.

The unscientific and highly unsatisfactory assumptions on which the present design of dams is based are getting more and more obvious. The assumption of linear stress distribution and the middle-third theory that presumed the absence of any tension stress in the dam was the beginning. Next came the Levy requirement that was, and still is, popular with European designers. Under this requirement, assuming again the linear distribution of stress, not only was the tension stress to be avoided, but at no point of the up-stream face of the dam was the compression stress supposed to be lower than the hydraulic water pressure at the corresponding elevation.

[41] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

[41a] Received by the Secretary January 2, 1934.

In the United States the sliding factor was favored for a long time as a controlling design assumption. The author introduces the shearing strength as a new and more desirable controlling design assumption and proposes the revival of the old Coulomb theory.

It would seem advisable to look at the entire matter in a more general way. First, it is to be realized that a gravity dam is a problem in the three-dimensional state of stress. The present limited engineering knowledge makes a proper handling of such problems very difficult. Consequently, the cardinal and necessary simplifying assumption is made in considering the vertical element of a dam as a two-dimensional stress problem. Other arbitrary assumptions must be added very carefully so as to keep the design reasonably close to actual conditions.

Fig. 12 illustrates the divergence between the actual stress, and the stress evaluated under the different design assumptions; it is only a schematic

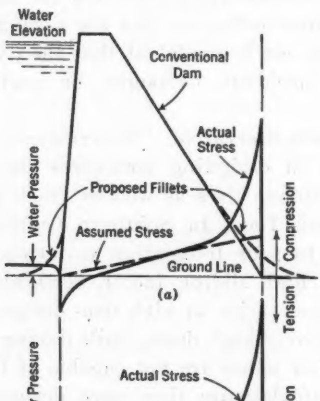

FIG. 12.—VERTICAL STRESS AND SHEAR AT THE GROUND LINE

representation and not to scale, either in the linear dimensions or in stress and shear values. Present knowledge does not permit the exact determination of the actual stress and shear, but is sufficient to make possible a schematic representation of the kind shown.[42]

[42] See, also, "Model of Calderwood Arch Dam" by A. V. Karpov and R. L. Templin, Members, Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1933, p. 1568.

An arbitrarily chosen vertical cross-section of a gravity dam is shown in solid lines in Fig. 12(*a*); the stress and shear at the foundation of this cross-section are to be considered, under full water load, assuming a two-dimensional state of stress. Fig. 12 (*a*) also shows the vertical stress distribution superimposed. In drawing this diagram it is assumed that the dimensions of the dam are such as to make it fulfill the requirements of the middle-third theory. The dotted line shows the vertical stress at the foundation evaluated under the assumption of linear distribution of stress and the resultant passing through the third of the base; consequently, the vertical stress equals zero at the heel of the dam. The solid line shows the actual vertical stress at the foundation of such a dam.

Fig. 12(*b*) shows the vertical stress distribution assuming such dimensions of the dam as to make it fulfill the Levy requirement. Again the dotted line shows the vertical stress evaluated under the linear stress assumption (the vertical stress equals the water pressure at the heel of the dam) and the solid line the actual stress. Fig. 12(*c*) shows the shear at the foundation. The author bases his theory on the full-base shearing strength, which is equivalent to the assumption of a uniform shear distribution. The dotted line represents the assumed shear distribution corresponding to Mr. Henny's proposal. The solid line represents the actual maximum shear.

From the study of these diagrams only one conclusion can be drawn; namely, that all three methods are unsatisfactory. As a matter of fact, there is not much difference between them; each one is arbitrary, none is proven, and an attempt to prove them will give results that are contrary to expectations. The author makes an attempt to disregard these facts and to introduce the non-existent imaginary constant shear. An attempt to make such an imaginary shear value a controlling factor in the design does not seem to the writer to be an improvement in present methods. The introduction of a safety factor based on such arbitrary shear value is rather meaningless since the maximum shear may be many times greater than the arbitrary assumed average shear.

There seems to be a pronounced tendency to forget the fact that in dealing with a large structure, such as a dam, the main question is whether or not the simplified assumptions that are applied to small test specimens may be transferred directly to the design of a dam. A concrete dam is an elastic structure which, depending on the foundation, rests on an elastic or a plastic base. To solve problems in the design of dams properly, recognition must be accorded that simple idea.

The modern tendency in engineering is to utilize the new researches in elastic and plastic problems and to apply them in accordance with modern developments in the theory of stress. As far as the design of dams is concerned, a new theory is not only useless, but may become dangerous, if it does not recognize the fact that, on account of the sharp changes in cross-section and the varying foundation conditions, considerable increase in local stress is possible.

The method proposed by Mr. Henny, in common with other methods, fails to develop the possibility of building up considerable stress concentration if there is an abrupt change in shape at the places where the dam proper meets the foundation. This increase of stress depends not only on the shape of the dam, but also on the elastic properties of the concrete and the foundation, and this stress may become very high on hard rock. The use of proper fillets to reduce this stress concentration is of considerable importance; on Fig. 12(*a*) such fillets are indicated by dotted lines. Any method that claims to be an improvement over the existing methods must reveal that fact clearly.

The importance of this consideration probably is well illustrated by the failure of the St. Francis Dam. This failure is attributed to the properties of the particular foundation rock to disintegrate when saturated with water and subjected to high stress. Such conditions were present and the dam failed. At the toe of the dam the compression stress could have been reduced from two to three times if proper fillets had been introduced. The removal of one of these unfavorable factors—the high stress—probably would have been sufficient to prevent failure or, at least, to make the failure less sudden and thus prevent loss of life and also to reduce property damage.

In order to have all these factors properly introduced in the design, it is necessary to eliminate the arbitrary assumptions and develop a theory that will cover the actual conditions with reasonable accuracy. From this point of view, the paper is of considerable value, since it stresses the present unsatisfactory design assumptions. The writer does not consider that the conclusions advanced in this paper are a step in the right direction. A proper scientific theory, even if rather complicated, is yet to be developed. A real improvement can be demonstrated only if such a theory will pass properly conducted model and prototype tests.

L. F. HARZA,[43] M. AM. SOC. C. E. (by letter)[43a].—By presenting an improved and more logical method of design against tension at the heel, the author has made a commendable contribution.

The subject of pore uplift, however, needs further treatment. The Woodard experiments indicate that internal pore pressure exists and that concrete can be fractured in tension by water under pressure entering at one point and escaping in opposite directions through the pores. The seepage water produces a differential pressure on each particle in the direction of seepage, because of the progressive loss in head that causes a greater pressure on the approaching than on the receding face of each particle, and, therefore, a tension if seepage is in opposite directions, as in the Woodard specimens.

An analogous condition does not exist in the pores of a dam, however. In this case, the water seeps in the same general direction, not upward or downward. Assuming approximately parallel and horizontal lines of seepage the condition becomes more nearly analogous to buoyancy; for example, if all the pores on the down-stream face could be closed by a water-tight surface, then static pressure would exist in each pore equal to its depth, H, below the

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[43a] Received by the Secretary January 15, 1934.

water surface. Bouyancy would be exerted within the concrete to an extent perhaps depending on some unknown function of the porosity. If the dam were composed of uncemented aggregate, full bouyancy would be effective, and the submerged weight of the material would be reduced by the full amount of its displacement, perhaps 80% or more of the gross volume; and yet, the pore space might be only 20%, or much less, of the volume. In this case the surface on which uplift is effective is not a plane cut through the material, but an irregular surface following entirely between the particles.

Likewise with cemented aggregates it may be argued reasonably that the internal bouyancy exceeds the proportion indicated by the porosity. A tension fracture of the material would expose a rough granular surface representing the weakest surface between the particles and not a plane cut through the pieces of aggregate. Now, if the water-tight down-stream face were to be removed, one could reasonably assume an average bouyancy of at least one-half the former, varying from the former value at the up-stream face to zero at the down-stream face, but always greater than that indicated by the porosity.

If the foregoing reasoning is correct, a similar effective proportionate pressure area much greater than the porosity, might assist to explain the apparent inconsistencies of the Woodard experiments in the author's Table 7. The principal inconsistency is more apparent than real. It results because the experiments should be divided in two groups. In the upper group in Table 7, the porosity increases upward as the specimens approach neat cement because impervious sand is being replaced with pervious cement matrix, and yet the strength increases. In the lower group, porosity is increasing downward because of reduction in the cement and is concurrent with reduction in strength. Even with this grouping, quantitative deductions of the laws of change are not possible, perhaps because of the few points in each group and the possible unequal distribution of stress over the cross-section. Even after discounting the results by possible unequal distribution of stress, they indicate clearly, however, that pore pressure is exerted over a much larger proportion of the cross-section than that represented by porosity, especially in the leaner mixes.

The usual assumption that uplift pressure decreases uniformly from heel to toe in a triangular manner presupposes uniform opening of the joints from heel to toe. The greater pressure toward the toe, however, would tend more nearly to close up the joints and increase their resistance, or to reduce the area subject to uplift as assumed by the author. In either case, this would tend toward a trapezoidal or perhaps parabolic rather than triangular variation of water pressure from heel to toe. This might be offset in net effect by silting of the up-stream face.

The author assumes, in Table 6, a buoyancy at the heel of three times, and, at the toe, of twice, that represented by porosity. This is perhaps enough for good concrete in the mass between construction joints, but it is not clear whether the author also intends it to apply to construction joints assuming good workmanship. From the Woodard experiments it would seem at least doubtful whether it can allow for more than the porosity.

The most serious problems of uplift must result from: (1) Lack of intimate contact along construction joints; (2) along the contact with the foundation; (3) bedding planes in stratified rock below the foundation level; or (4), local uplift communicated to isolated areas under the dam through fissures or joints in the rock. All these conditions are defects either in Man's or Nature's part of the structures and cannot yield to any rational basis of analysis. Apparently, allowance for them must forever depend entirely on judgment gained by experience.

Lack of intimate contact at construction joints can readily occur because of the necessity of placing one layer of material, subject to initial volume changes, on a layer already set. Creeping or even warping of the new layer over the old one to an extent sufficient to break the bond is not difficult to imagine as the result of internal stresses set up by these relatively different tendencies to volume change. Such condition would not be revealed by the Bull Run test specimens.

The writer is not quite confident that increase in vertical loading can be assumed to decrease the effective uplift pressure area as indicated in Table 6 in the case of such defective contact. It seems unlikely that a joint once broken can be restored to such intimate contact by pressure as to exclude water. To the extent that such pressure would tend to reduce the opening of the joint, it would cause a greater initial loss of head of seepage water entering the joint and thus would reduce uplift pressure rather than area.

May the writer have the temerity to suggest a radical departure in dam design and construction to eliminate uncertainty from pore and joint pressure and the progressive changes in composition, in some cases approaching disintegration of concrete, due to seepage? Stainless steel and some other metals practically free from corrosion are now available, the former at a cost of about 40 cents per lb. Such metal of a thickness to weigh about 1 lb per sq ft might be rolled with relatively small corrugations, say, 1 in., in one direction and then with large corrugations, say, 3-in. spacing, in the other direction to make the sheets elastic both ways. Such sheets could be used as lagging or alternately inside the lagging on the up-stream face, welded or soldered at the joints and perhaps with special U-shaped seams at all vertical and horizontal construction joints and with frequent spot-welded strips to embed in the concrete and insure adhesion. Such an impervious surface would not cost more than 1 ft or 2 ft of extra thickness of concrete, a negligible amount for a large dam. Its effectiveness should be complete, except where most needed, at the joint with the foundation rock. This problem might be minimized, but never entirely eliminated, by a heel apron similarly surfaced.

The writer fully agrees with the author's general argument for shear factor instead of sliding factor for dams on non-stratified foundations. Mr. Henny admits that sliding factor must still be considered when a foundation contains horizontal stratification. The fact should here be pointed out that the extensive regions of stratified rock, forming a considerable proportion of the earth's surface, do not permit general abandonment of the sliding

factor method of analysis, a fact which should be emphasized to avoid possible future misapplications of the author's very valuable contribution to the art. Even in stratified rock the contact surfaces are frequently undulating so that shear as well as sliding will enter in variable amount.

EDWARD GODFREY,[44] M. AM. SOC. C. E. (by letter) [44a].—The vast difference in stability of the Boulder Dam and other large dams designed at earlier dates, particularly some that have failed, is ample demonstration of the fact that the design of dams has departed from old standards. Mr. Henny's paper is timely and important, particularly because it comes at a time when engineers are "rethinking" the design of dams. The emphasis placed upon, and the prominence given to, under-pressure in dams in Mr. Henny's paper is quite significant, in view of the fact that not so far in the past it was practically impossible to find any mention of this in the literature of dams.

Mr. Henny emphasizes the importance of considering shear as the prime factor in designing a concrete dam for strength. He also points out the difference in shear strength in concrete that is in tension normal to the shearing plane and concrete that is in compression. This leads straight to the question of tension in the concrete, which, in turn, brings up interior or under-pressure in the body of the dam, or the forces that lift it from its foundation.

A factor that has been neglected in considering the stability of a dam is flotation. Mr. Henny states:

"When it became evident that failure of solid gravity dams was generally accompanied by down-stream mass movement and that this was facilitated if not induced principally by penetration of pressure water at the base of the dam, or in the foundation, two new factors were taken into account, namely, uplift and sliding."

The down-stream mass movement was never so conspicuously exemplified as in the case of the St. Francis Dam. Great masses of solid concrete, weighing thousands of tons, were moved down the valley for thousands of feet. This is flotation pure and simple. It seems to be a prevailing notion that flotation occurs only when a body is submerged or partly submerged—in other words, when there is water on all sides of it. In the past, engineers who ignored under-pressure, or argued against its inclusion in designing, did not fail to consider flotation or loss of weight of masonry when tail-water was present. The depth of the tail-water was the only head recognized, when the fact is that the greater the head of tail-water, the safer the dam, up to complete safety with tail-water level with the main body (barring flow).

Water on the vertical sides of a submerged vessel has no influence in causing flotation. Water on the down-stream slope of a dam helps to hold it down, rather than to lift it. A submerged body is subject to flotation solely by reason of the pressure of water (or difference in pressure) below the body—in other words, the vertical component of pressure on all surfaces in contact with the water.

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[44] Structural Engr., Pittsburgh, Pa.

[44a] Received by the Secretary January 19, 1934.

In a dam in which the water is in contact with the up-stream surface and has access to the base, unrestricted, the flotation is just the same as if it were completely submerged; but the horizontal forces are quite different.

In Fig. 13, A represents the cross-section of a concrete dam in which the base is 0.84 of the height; B represents the horizontal force of water on the up-stream side; C, the upward pressure under the base, full head at N tapering to zero at M; and D, the upward pull against the foundation, forces induced by the water pressure, B, tending to rotate the dam about M.

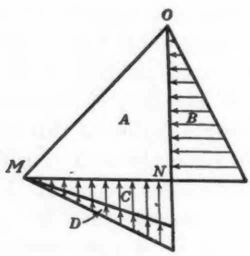

FIG. 13

In a dam of these proportions, the sum of the forces, C and D, exactly balances the weight of the concrete, A. If it were not for adhesion to the foundation, or if the foundation were not below ground level, this dam would be just on the verge of floating away, because it exerts no pressure on its base, except that which water balances exactly. A dam subject to analogous forces, with a base less than 0.84 of its height (say, 0.65, as in the St. Francis Dam), has every condition favorable for complete flotation. The reaction of the masonry on the foundation is less than the possible pressure of the water. Here is flotation with the addition of a gigantic force to push the dam down stream and a complete explanation of the down-stream mass movement referred to by Mr. Henny.

Under-pressure can no longer be reasoned away. The mere setting up of tests that seem to show its non-existence in some cases has no weight in destroying or controverting the great volume of test and experience that prove its existence in all kinds of cases.

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THE SURVEYOR AND HIS LEGAL EQUIPMENT

Discussion

By Messrs. C. H. Eiffert, and Verne G. Sanders

C. H. Eiffert,[98] M. Am. Soc. C. E. (by letter)[98a].—There are too many young engineers and surveyors who seem to believe that land surveying is merely a matter of turning angles and measuring distances. They are apt to place a tack in a hub with the greatest of precision but, at the same time, overlook or ignore evidences of original corners and boundary lines which, although they may not have been located accurately, are nevertheless the indicators of the true locations of the original lines. The writer would like to recommend this paper especially to these younger members of the profession.

It is even more difficult to convince those outside the profession that land surveying is not merely a matter of running bearings and distances given in a deed or on a plat. True, it is somewhat disconcerting, and humbling to the pride, of one who has been trained in accurate work, to find that, after measuring very precisely a recorded distance, and carefully placing the tack, it is necessary to adjust the distance 5 or 6 ft, more or less, and change the bearings similarly in order to conform to the location of a rail fence, or to that indicated by a decrepit looking bearing tree, or perhaps only the stump of such a tree. "Such is the life" of the real surveyor, however, and such difficulties should be no deterrent against "keeping the chain level, and stretched tight" and otherwise doing accurate work, because once the true line is found or re-established the accurate bearings and distances are necessary to guard against future difficulties.

It is rarely indeed that measurements made to-day with steel tape, spring balance, and correction for temperature, will agree closely with those made 50 or 75 years or more ago when land was cheap, when link chains with

Note.—The paper by A. H. Holt, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. Ray H. Skelton, William Bowie, R. Robinson Rowe, and Walter H. Dunlap; December, 1933, by Chester Mueller, Assoc. M. Am. Soc. C. E.; and January, 1934, by Messrs. Clarence T. Johnston, A. F. Harley, and Lynn Perry.

[98] Chf. Engr., The Miami Conservancy Dist., Dayton, Ohio.

[98a] Received by the Secretary January 2, 1934.

600 or 800 wearing surfaces were used, and when the sight compass, not the transit, was the instrument utilized.

The writer has been told by those who claimed to be eye-witnesses that on the Western prairies surveys were made by steering with a pocket compass and measuring distances by counting the revolutions of the wagon-wheel, and that stones were simply dumped out of the wagon at the section and quarter corners. Nevertheless, right or wrong, these stones became the corners, and after the surveys had been accepted by the Government it became the legal duty of the surveyors retracing the lines to use these stones or the lines of occupation which had been etstablished in accordance with them. On such original surveys carelessly and inaccurately made, the township or other plats filed in the Government Land Office will show the miles to be 80.00 chains in length and the half-miles, 40.00 chains, and the bearings will be recorded within the required limits of accuracy; yet a resurvey will show them to be grossly inaccurate, and agreement with any of the recorded bearings or distances will be purely accidental.

A resurvey of such lines may be made with the greatest accuracy and exactly in accordance with the plat and yet be entirely wrong and inadequate if it ignores existing evidence showing where the corners were placed originally.

Too many times the surveyor, if hired by an interested party to relocate a disputed boundary line, assumes a hostile attitude toward the other party or parties in the dispute. It should be remembered that in most lawsuits arising from boundary disputes, unless extremely valuable property is involved, even the winner loses. If the surveyor can bring about the peaceful settlement of such a dispute he will be instrumental in saving money for both parties. In most cases this can be done if, after he has obtained all the facts in the case, he will explain them to the disputants and show them what the decision would probably be if the matter were taken into Court.

The writer knows of an instance in which a surveyor was employed to re-run a line in the northern woods. This he did, cutting a nice straight line and placing concrete monuments thereon. All unbeknown to him the true line, sufficiently marked so that it could be identified without a doubt, was peacefully slumbering in the woods off to one side. Other surveyors came in to lay out summer resorts, tied them to the newly monumented line, and had the plats recorded. Later, an adjoining timber owner wishing to have his timber estimated sent in a cruiser for that purpose. This man, familiar with the ways of the woods, located the original lines without difficulty. An expensive and extensive lawsuit resulted.

The re-establishment of lost or obliterated corners is an important matter and is not a job for the novice any more than are important matters in other professions. It requires common sense and good judgment and a knack of observing signs which are acquired only after considerable experience.

The U. S. General Land Office has issued a pamphlet entitled "Restoration of Lost or Obliterated Corners and Subdivision of Sections." This should be in the possession of every surveyor who may be called upon to do such work.

VERNE G. SANDERS,[99] ESQ. (by letter)[99a].—In the days when the white man started to settle the Colonies, land was plentiful, owners were few, and precise boundaries were of no importance. When towns and cities began to come into existence, neither surveyors nor land owners could visualize the future values of property, and the litigation to be involved in such a few short years. At that time there was an excuse for the poor deed descriptions and the insecure methods of boundary marking.

To-day, with the history of the almost unbelievable increase in real property values, one can look into the future with some comprehension. Now, there is no excuse for members of the Engineering Profession if they do not do all in their power to further a program of permanent establishment of property boundaries, be they across the arid desert or in the most congested districts of large cities.

The legal problems of boundary re-establishment that Professor Holt gives in his paper can be simplified, and may even be eliminated for posterity through the proper use of triangulation for controlling all surveys. By this method of control, permanent co-ordinates can be placed on each angle-point of all boundaries and filed with the deeds. Then, when re-establishment becomes necessary, all surveys for any one boundary will start at the same point, the location of which is controlled by the most accurate method of surveying known, namely, triangulation.

Several cities now use triangulation in many ways. Engineers in Los Angeles, Calif., have for many years felt the need of a precise method of control, but only in 1933, through the organization of a committee with a representative from each of the governmental agencies responsible for surveys and planning, have they been able to get a program worked out.

The first step in this program is a primary first-order net of triangulation connected to five points of the main triangulation of the U. S. Coast and Geodetic Survey. This net covers the southwestern part of Los Angeles County, an area of nearly 1 000 sq. miles, and is planned so that adequate main control is furnished over the entire City of Los Angeles.

The computations of all triangulation and traverse will be made on North American 1927 Datum. The plane system of co-ordinates as used by the Bureau of Water Works and Supply of Los Angeles for the last ten years has been adopted as a common system by all the organizations involved.

As the various steps of break-down and traverse proceed to completion, there will be one or more first-order points available for every property or street survey to be made in the County or City of Los Angeles. It is confidently expected that the legislative bodies of the City and County will pass the necessary ordinances to have all subsequent sub-divisions, resurveyed, and adjudicated boundaries properly tied in and co-ordinated.

In the future this will save untold sums to property owners through the elimination of litigation, and untold sums to the taxpayer by permanently fixing the location of street and highway center lines.

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[99] Chf. Computer, Joint Committee on Survey Control, Through Triangulation, County and City of Los Angeles, Los Angeles, Calif.

[99a] Received by the Secretary January 2, 1934.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WATER-BEARING MEMBERS OF ARTICULATED BUTTRESS DAMS

Discussion

By Messrs. R. A. Sutherland, P. Wilhelm Werner, Charles P. Williams, and Paul Baumann

R. A. Sutherland,[4] Assoc. M. Am. Soc. C. E. (by letter)[4a].—Articulated buttress dams are applicable to a wide variety of foundation conditions, showing in many cases greater economy and in almost all cases greater security than gravity dams. In certain cases, the choice of this type may be indicated by considerations not entirely economic (such, for example, as the

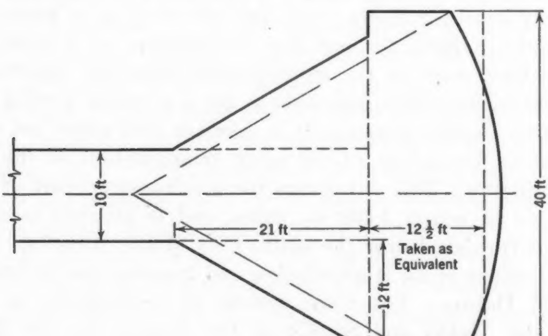

Fig. 7.—Section of Round Head at Crest Level.

likelihood of earth movements), and it is in such cases that the value of the author's results is greatest, in that they then enable the parts of the structure to be proportioned in the best manner.

In other cases, a comparison between the buttress type of dam and other types (in particular the gravity dam), may be the deciding consideration, and

Note.—This paper by Hakan D. Birke, Jun. Am. Soc. C. E. (now Assoc. M. Am. Soc. C. E), was published in September, 1933, *Proceedings* This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[4] Wellington, New Zealand.

[4a] Received by the Secretary December 5, 1933.

the writer recently had occasion to make such a comparison. He was impressed by the fact that the shape of the dam site largely influences the relative merits of the two types on the score of economy. In Fig. 5, certain heights are shown concerning which the author states "the best economy will be obtained by omitting the deck entirely and using the round-head type." The writer will confine his remarks to a comparison between a round-head type of dam and a gravity dam. The dimensions of the round-head type for comparison are shown in Fig. 7.[5]

In Table 2 one buttress and round head is compared with a 40-ft slice of a gravity dam with total batter equal to 0.8. The batter of the buttress is

TABLE 2.—COMPARISON OF BUTTRESS AND ROUND-HEAD DAM, WITH A 40-FOOT SLICE OF A GRAVITY DAM

Height of buttress, in feet	Volume of buttress, etc., in cubic feet	Volume of gravity slice, in cubic feet
30	31 260	14 400
60	74 080	57 600
90	130 700	129 600
120	202 200	230 400

0.6 both up stream and down stream; the crest thickness is 10 ft, increasing at the rate of 1 ft for every 30 ft of height. In both cases, no consideration is given to additional material in the crest of dam, it being assumed that the extra material in the crest of the gravity dam will cost about the same as the roadway, etc., on the crest of the buttress dam. The volume of the haunches and the buttress proper has been computed by the prismoidal rule.

It will be seen that if the height of the dam were uniform throughout, economy of material is shown only for structures higher than 95 ft, and economy of cost only for even greater heights. No dam, however, is of uniform height throughout, and economy in the entire dam will depend greatly on whether a large part of it is of a considerable height, or whether it spans a relatively narrow gulch with long flanks of lower height; in other words, economy will depend on the value of n as used by the writer elsewhere[6] as an index of the shape of the site. Using a procedure described elsewhere[6] the writer was able to develop a simple expression to determine the height above which the buttress dam showed a given degree of economy, as compared with a gravity dam. However, this was only for the assumption of parallel-sided buttresses, as the consideration of tapered buttresses led to expressions which are not readily integrable.

On grounds similar to the foregoing comparison with a gravity dam, the writer would suggest that the limits shown in Fig. 5 (limits above which the round-head type should be used), while perfectly legitimate in considering an individual buttress, should be modified in considering the entire dam, for the reason that if the round head is decided on, the entire dam must generally be of that type. The round-head buttress does not lend itself to combination with the slab type, nor with a gravity dam. The writer would

[5] *Engineering News-Record*, February 11, 1932, p. 212, Fig. 3.

[6] "Some Aspects of Water Conservation," *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 157.

be glad to know whether the author has considered the desirability, of reducing buttress spacing toward the flanks of the dam, particularly with the round-head type. With this type there seems to be no inherent reason for a uniform spacing, if economical considerations dictate otherwise.

P. WILHELM WERNER,[7] ASSOC. M. AM. SOC. C. E. (by letter)[7a].—The economic principles involved in this study are of importance for structures other than buttress dams. The author has assumed the factor of distribution of shear, j, to be constant. It is questionable, however, whether this is true for the haunches, the depth, d, of which is great in relation to their span, a. Investigations made by the writer seem to indicate that for such beams the shear distribution is dependent to a considerable degree, on the ratio, $\frac{d}{a}$, although sufficient investigations are not yet available to give a definite quantitative answer.

In his determination of $\frac{\partial V_h}{\partial b}$, Mr. Birke has included in the cost of the haunch a steel area, R_2, for the pull due to the friction between the deck and the haunch surfaces to allow for a possible movement in the deck (Equation (31)). As far as the writer is aware, no corresponding steel area is included in the cost of the deck (Equation (39)), where it is evident that there must be a reactive pull of the same magnitude due to the pull in the haunches. Probably, there is some reason for omitting the corresponding steel in the deck, and, if so, it would be interesting to know it.

Mr. Birke has investigated the relative economy of the deck and the haunches, without going into the economy of each member separately, except to use stresses that are customary for the type of structures under considera-

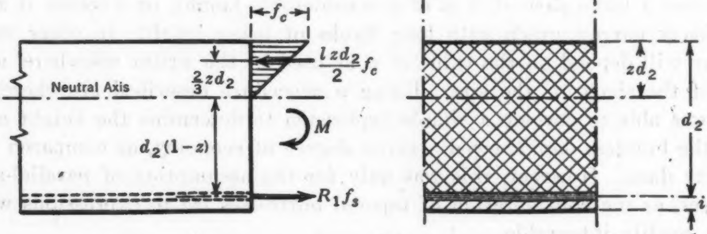

FIG. 8.

tion. It is well known that, in a slab in flexure, there is a certain economic relation between the steel and the concrete. It would seem logical in this connection to consider this relation, especially in view of the fact that for the type of structures in question the space available for the depth of the deck is not restricted by influences other than those of economy. The point may also be of interest in view of the present trend toward revising standard specifications for maximum allowable stresses in reinforced concrete.

[7] Stockholm, Sweden.

[7a] Received by the Secretary December 5, 1933.

Considering the slab shown in Fig. 8, and using the nomenclature of the paper as far as possible, the following well-known relation is obtained,

$$\frac{f_s}{f_c} = \frac{E_s}{E_c}\,\frac{1-z}{z} \qquad (61)$$

or, with $\frac{f_s}{f_c} = \beta$ and $\frac{E_s}{E_c} = n$,

$$\beta = n\,\frac{1-z}{z} \qquad (62)$$

The equilibrium condition for the moments gives, $M = z\,(3 - z)\,\frac{l d_2^2}{6}\,f_c$; from which,

$$d_2 = \sqrt{\frac{6\,M}{l\,f_c}}\;\frac{1}{\sqrt{z\,(3-z)}} \qquad (63)$$

The equilibrium condition for horizontal forces gives, $\frac{l\,z\,d_2}{2}\,f_c = R_1\,f_s$; from which,

$$R_1 = \frac{l\,z\,d_2}{2\,\beta} \qquad (64)$$

The cost of the concrete, C_{cd}, is determined by Equation (65) (the volume occupied by the steel and the steel cover, i_2, which is practically constant, are not taken into account):

$$C_{cd} = d_2\,l\,L\,D_c \qquad (65)$$

and the cost of the steel, C_{sd}, by the equation:

$$C_{sd} = (1 + e_2)\;\frac{z\,d_2}{2\,\beta}\,l\,L\,D_s \qquad (66)$$

Thus, the total cost, C_{td}, of the slab, except the cost of the forms and other practically constant items, is:

$$C_{td} = C_{cd} + C_{sd} = d_2\left[1 + (1 + e_2)\,\frac{D_s}{D_c}\,\frac{z}{2\,\beta}\right] l\,L\,D_c \qquad (67)$$

Letting,

$$\kappa = \frac{1 + e_2}{2\,n}\,\frac{D_s}{D_c} \qquad (68)$$

and substituting the values of β (Equation (62)), and of d_2 (Equation (63)), in Equation (67):

$$C_{td} = \frac{1}{\sqrt{z\,(3-z)}}\left(1 + \kappa\,\frac{z^2}{1-z}\right)\sqrt{\frac{6\,M}{l\,f_c}}\;l\,L\,D_c \qquad (69)$$

or, if $Z = \frac{1}{\sqrt{z\,(3-z)}}\left(1 + \kappa\,\frac{z^2}{1-z}\right)$,

$$C_{td} = Z\,\sqrt{\frac{6\,M}{l\,f_c}}\;l\,L\,D_c \qquad (70)$$

Equation (70) is evaluated in Fig. 9, which shows the cost of the slab for various ratios, $\frac{D_s}{D_c}$, at constant value of f_c.

The value of z that will make C_{td} a minimum is controlled by the condition,

$$\frac{\partial C_{td}}{\partial z} = 0 \qquad (71)$$

By differentiating the expression for C_{td} in Equation (69) (in which, f_c is assumed constant), substituting in Equation (71), and reducing, the fol-

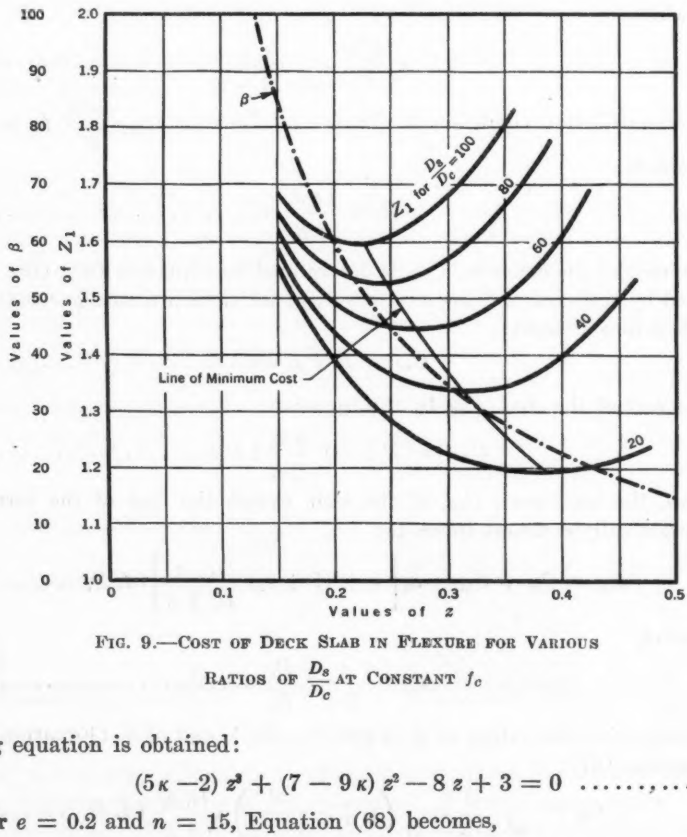

FIG. 9.—COST OF DECK SLAB IN FLEXURE FOR VARIOUS RATIOS OF $\frac{D_s}{D_c}$ AT CONSTANT f_c

lowing equation is obtained:

$$(5\kappa - 2) z^3 + (7 - 9\kappa) z^2 - 8z + 3 = 0 \qquad (72)$$

For $e = 0.2$ and $n = 15$, Equation (68) becomes,

$$\kappa = 0.04 \frac{D_s}{D_c} \qquad (73)$$

For different values of the ratio, $\frac{D_s}{D_c}$, the most economical ratio, $\beta = \frac{f_s}{f_c}$, at constant f_c, can now be computed by solving Equation (72) for z and substituting these values of z in Equation (62). The result is shown in

Fig. 10, for values of the ratio, $\frac{D_s}{D_c}$, between 20 and 100, which probably represent practical limits of the ratio between the costs of steel and concrete.

Taking $\frac{D_s}{D_c} = 60$, for instance, it is seen from Fig. 9 that the most economical ratio between the stresses, f_s and f_c, is $\beta = 40$. With a concrete stress of $f_c = 650$ lb per sq in., as used by the author, the steel stress would then be $f_s = 24\,000$ lb per sq in. For $\frac{D_s}{D_c} = 40$, the corresponding values would be $\beta = 34$ and $f_s = 22\,000$ lb per sq in. These values exceed, greatly, the allowable steel stress of 16 000 lb per sq in. used, and are in any case out

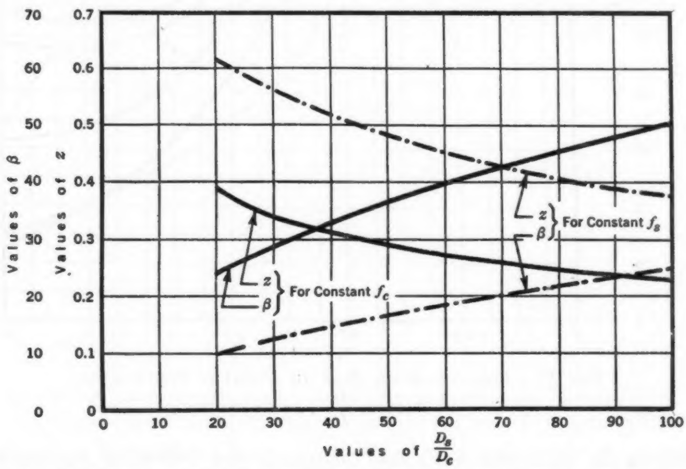

FIG. 10.—ECONOMIC DIMENSION AND STRESS RELATION OF DECK SLAB IN FLEXURE AT CONSTANT f_c OR CONSTANT f_s

of the question in structures of the type under consideration, especially in view of the necessity of avoiding cracks in the concrete. It is, therefore, of greater interest in this connection to investigate the problem under the assumption of a constant steel stress.

By substituting the value of f_c (Equation (62)), in Equation (69), C_{td} becomes,

$$C_{td} = \frac{1}{z}\sqrt{\frac{1-z}{3-z}}\left(1 + \kappa\,\frac{z^2}{1-z}\right)\sqrt{\frac{6M}{l f_s}}\, n\, l\, L\, D_c \quad \ldots\ldots\ldots (74)$$

in which, $\frac{1}{z}\sqrt{\frac{1-z}{3-z}}\left(1 + \kappa\,\frac{z^2}{1-z}\right) = Z$

This equation is evaluated in Fig. 11, showing the cost of the slab for various ratios, $\frac{D_s}{D_c}$, at constant f_s.

The value of z that will make C_{td} in Equation (74) a minimum is obtained under the condition expressed by Equation (71). By differentiating the expression for C_{td} in Equation (74) (in which, f_s is assumed constant), and

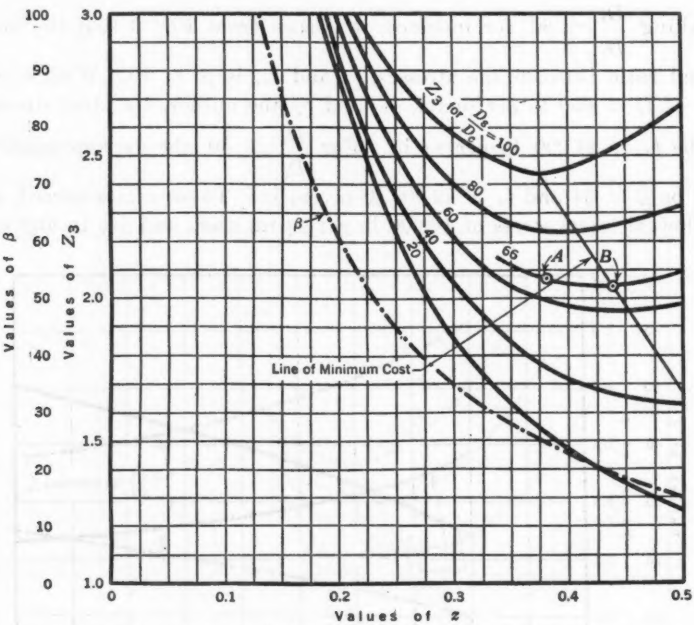

FIG. 11.—COST OF DECK SLAB IN FLEXURE FOR VARIOUS RATIOS, $\frac{D_s}{D_c}$, AT CONSTANT f_s

substituting in Equation (71) and reducing, the following equation is obtained:

$$(1 - 2\kappa) z^3 + (3\kappa - 4) z^2 + 6z - 3 = 0 \quad \dots\dots\dots\dots(75)$$

Using the value of κ given in Equation (73) the most economical ratio, $\beta = \frac{f_s}{f_c}$, at constant f_s, can now be obtained by solving Equation (75) for z and substituting the resultant values of z in Equation (62). The result is shown by dash-dotted lines in Fig. 10.

In the typical example introduced by Mr. Birke, with $f_s = 16\,000$ and $f_c = 650$ lb per sq in., the value of β is $\beta = 24.6$. Thus, for $\frac{D_s}{D_c} = 66$, the point, A, in Fig. 11, is obtained. The minimum cost is obtained for $\beta = 20$, corresponding to the point, B, in the diagram. It is seen that the values of the stresses used by the author very nearly give the best economy with the ratio, $\frac{D_s}{D_c} = 66$. It should be noted that using a lower value of β than about 20 (Point B, Fig. 11) will only increase the cost.

Charles P. Williams,[8] M. Am. Soc. C. E. (by letter)[8a].—Only within the last few years, have attempts been made to determine directly the most economic proportions of the class of structures treated in this paper, and economy of design has been secured only by the comparison of several designs, in which many of the controlling dimensions were selected arbitrarily. It has been the custom to select an "out-to-out" length of deck span equal to the clear spacing of buttresses. The author demonstrates that, in many cases, this is not economical, and, moreover, that at a determinable distance below the top of the dam, it is more economical to omit the deck altogether, substituting for the "articulated buttress" type of dam, the "round-head buttress" type, devised by the late Fred A. Noetzli, M. Am. Soc. C. E.

While Mr. Birke's analysis is presented in a logical and ingenious manner, a few of his assumptions are in error. His analysis of haunch stresses is based on the commonly used formulas for a beam of uniform depth, which, when a unit width of beam is considered, are as follows:

$$j = 1 - \frac{k}{3} = 1 - \frac{n f_c}{3 (f_s + n f_c)} \qquad (76)$$

$$\tau \text{ (max.)} = \frac{S}{j\, d} \qquad (77)$$

$$f_c = \frac{2 M}{j\, k\, d^2} \qquad (78)$$

and,

$$R_1 f_s = \frac{M}{j\, d} \qquad (79)$$

in which, in addition to the notation of the paper: n = the ratio of the coefficient of elasticity of steel to that of concrete; and k = a coefficient such that $k\, d$ equals the distance from the neutral axis to the extreme compression element, measured in the direction of the applied external load.

Equations (76), (77), (78), and (79) are applicable only when the tensile steel and the extreme compression filament are each perpendicular to the direction of the external load. The late William Cain, M. Am. Soc. C. E., has given[9] an approximate solution of the case, in which either or both the tensile steel and the extreme compression filament are inclined to the direction of the external load.

In the following discussion, the terms, shear, unit shear, and total shear, signify shear in the plane, *A-A*, Fig. 12, unless otherwise expressly stated.

In a beam having parallel tensile and compressive elements, each perpendicular to the direction of the external load, the unit shear in any plane parallel to the direction of the loading (assuming the concrete to carry no tension) is uniform on the tensile side of the neutral axis, and is the maximum unit shear in the section. On the opposite side of the neutral axis, the

[8] Cons. Engr., Los Angeles, Calif.

[8a] Received by the Secretary December 13, 1933.

[9] "Earth Pressures, Retaining Walls, and Bins," by William Cain, p. 239 *et seq.*, N. Y., John Wiley & Sons.

unit shear decreases at a rate proportional to the square of the distance from that axis, becoming zero at the extreme compression filament.

In the case of a beam having the tensile element perpendicular to the direction of the external load, but having its compression face inclined thereto, making the angle, ϕ, with the tensile element, as shown in Fig. 12, it can be shown that the factor, j, the unit shear, τ_1, at all points on the tension side

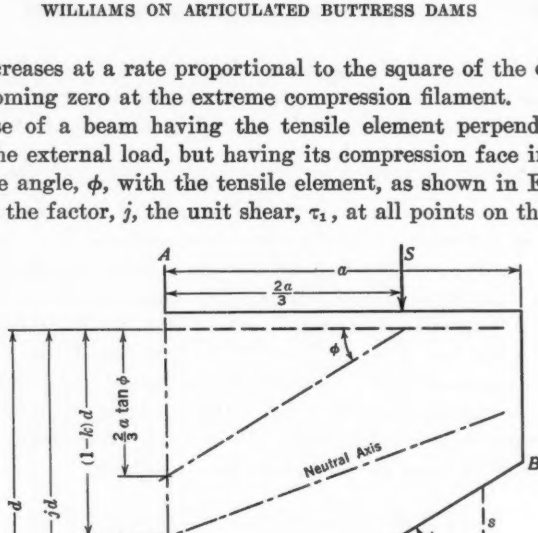

FIG. 12.

of the neutral axis, the unit shear, τ_2, on the opposite side of the neutral axis, at a point distant y therefrom, and the unit shear, τ_3, at the extreme compression filament, are given by the following equations:

$$j = 1 - \frac{k}{3} = 1 - \frac{n\, f_c \cos^2 \phi}{3\,(f_s + n\, f_c \cos^2 \phi)} \quad \ldots\ldots\ldots\ldots(80)$$

$$\tau_1 = \frac{1}{j\, d}\,(S - \frac{M}{d} \tan \phi) \ldots\ldots\ldots\ldots\ldots\ldots(81)$$

$$\tau_2 = \tau_1 - \frac{\tau_1}{k^2\, d^2}\, y^2 + \frac{2\, M \tan \phi\, y}{k^2\, j\, d^3} \quad \ldots\ldots\ldots\ldots(82)$$

and,

$$\tau_3 = f_c \sin \phi \cos \phi = \frac{2\, M \tan \phi}{k\, j\, d^2} \quad \ldots\ldots\ldots\ldots\ldots(83)$$

The values of τ_1 and τ_2, in Equations (81) and (82), are approximations, based on the assumption that j is constant throughout a strip of infinitesimal width at the section, *A-A*, considered. The total shear at the section is S. The error in the approximation may be indicated by computing the total shear from the equations for unit shear. Designating by S_c, the total shear,

computed from Equations (80) and (81), the following equation is obtained:

$$S_c = (1 - k)\, d\, \tau_1 + \int_0^{kd} \tau_2\, \partial y \qquad (84)$$

Substituting for τ_1 and τ_2, their approximate values, performing the operations indicated, and reducing, the computed total shear is found to be:

$$S_c = S + \frac{1 - j}{j\, d} M \tan \phi \qquad (85)$$

Substituting for M its value, $\frac{2}{3}\, a\, S$, and reducing:

$$S_c = S \left\{ 1 + \left(\frac{1 - j}{j} \right) \frac{2\, a \tan \phi}{3\, d} \right\} \qquad (86)$$

Assuming $f_c = 650$ lb per sq in.; $f_s = 16\,000$ lb per sq in.; and $n = 15$, the value of j will range from 0.922 for $s = \tan \phi = 1$, to 0.992 for $s = 5$. For s ranging from 1 to 5, then, the value of $\frac{1 - j}{j}$ will range from 0.085 to 0.008. From Fig. 12, it will be observed that, $a \tan \phi < d$; hence, $\frac{2\, a \tan \phi}{3\, d} < \frac{2}{3}$. The error in the computed value of S_c, then, is less than 0.057 S when $s = 1$, and less than 0.005 S, when $s = 5$. For haunches in which $\frac{d}{a}$ is large, the error will be much less.

The distance from the neutral axis, corresponding to the point of maximum shear, is found from Equation (82) by placing the first derivative (as to y) of τ_2, equal to zero and solving for y. Performing these operations, it is found that the value of y at which the shear is a maximum, is:

$$y = \frac{M \tan \phi}{S - \frac{M}{d} \tan \phi} = \frac{M \tan \phi}{j\, d\, \tau_1} \qquad (87)$$

It may be found that the point, determined by this value of y, is outside the structure, in which case the maximum unit shear is τ_3, as given in Equation (83). If, however, the point falls within the structure, the value of y is substituted for y in Equation (82), and reducing, the maximum unit shear is found to be:

$$\tau_2\,(\text{max.}) = \tau_1 + \frac{(M \tan \phi)^2}{k^2\, j^2\, d^4\, \tau_1} \qquad (88)$$

The maximum unit shear may exceed greatly the unit shear on the tension side of the neutral axis.

At the point, P, in the section, A-A, Fig. 12, the compressive stress is the first principal stress. The direction of the maximum unit shear is at an angle of 45° with the direction of the principal stress. If, then, $\phi = 45°$, the maximum unit shear at P is in the plane, A-A. Between P and the neutral axis the maximum unit shear is approximately in that plane. If, however, ϕ is greater or less than 45°, the unit shear in the plane, A-A, at the point, P,

will not be the maximum unit shear at that point, and the same will be true approximately for other points in the plane, A-A, between P and the neutral axis.

From the foregoing it might appear that there is greater danger of failure from excessive shear on the compression side of the neutral axis than on the tension side. This is not the case, however, since usually failure occurs from diagonal tension, incident to the shear, and not from direct shear.

The compressive forces below the neutral axis may be resolved into two components, one normal to the plane, A-A, and causing compression, and one parallel thereto and causing shear in the direction of the plane, and also in the direction perpendicular to it. The variation of shear below the neutral axis is due to the compressive forces, which are assumed to increase uniformly from the neutral axis to the extreme filament. The uniform increase in the normal component results in a decrease in the unit shear, in proportion to the square of the distance from the neutral axis, and the uniform increase in the parallel component results in a uniform increase in the unit shear. Any net increase in the unit shear is due to the excess of the effect of the parallel component over that of the normal component. Now, it can be shown that these compressive forces do not cause diagonal tension, hence any increase in shear below the neutral axis will not result in diagonal tension. The unit shear above the neutral axis, as given by Equation (81), may be taken as a measure of the diagonal tension.

The author determines the depth of the haunch, d_1, by Equation (7), assuming values for j_1 and τ. He then determines the area of steel, R_1, required to resist the bending moment, by Equation (30), assuming a value for f_s. Not more than two of the three variables, j, τ, and f_s can be assumed, which is demonstrated as follows: In order to simplify the equations, assume a beam with parallel tension and compression faces; that is, $\phi = 0$; then,

$$\tau = \frac{S}{j\,d} \quad \text{.........................(89)}$$

and,

$$M = \frac{f_c}{2}\,kd\,jd = \frac{3}{2}\,f_c\,(1 - j)\,jd^2 \quad \text{...............(90)}$$

From Equation (89),

$$d = \frac{S}{j\,\tau} \quad \text{.........................(91)}$$

and from Equatioin (90):

$$d_2 = \frac{2\,M}{3\,f_c\,(1 - j)\,j} = \frac{4\,a\,S}{9\,f_c\,(1 - j)\,j} \quad \text{.............(92)}$$

Hence,

$$\frac{S}{\tau^2\,j} = \frac{4\,a}{9\,f_c\,(1 - j)} \quad \text{.........................(93)}$$

and,

$$f_c = \frac{4\,a\,\tau^2\,j}{9\,(1 - j)\,S} \quad \text{.........................(94)}$$

The value of j is determined by the equation:

$$j = 1 - \frac{n f_c}{3\,(f_s + n f_c)} \quad \text{.....................(95)}$$

from which may be derived the equation:

$$f_c = \frac{3\,(1 - j)\,f_s}{n\,(3j - 2)} \quad \text{.......................(96)}$$

Substituting this value of f_c for f_c in Equation (94):

$$\frac{3\,(1 - j)\,f_s}{n\,(3j - 2)} = \frac{4\,a\,\tau^2\,j}{9\,(1 - j)\,S} \quad \text{...................(97)}$$

The variables in Equation (97) are j, τ, and f_s. Therefore, when any two are assumed, the other is fixed.

In Equation (55), Mr. Birke assumes the load per unit area of deck to be the water-load only. The haunch must carry, not only the water-load, but also the normal component of the weight of the deck and of the haunch itself. For an Ambursen dam, having a buttress spacing of 22 ft, center to center, the normal component of the weight of the deck varies from 0.16 W, at 30 ft below the top, to 0.05 W, at a depth of 180 ft. The weight of the deck is a function not only of the depth below the water surface, but also of the span. An increase in the dimension, b, will result in a decrease of the thickness of the deck, and its consequent weight; however, the weight of the haunch will be increased.

Referring to Equation (44), the author states that "for a constant head, the ratio, $\frac{b}{L_2}$, will be independent of the clear buttress spacing." For this to be true, $\frac{c}{L_2}$ must be constant. It is constant as to the water-load, but not as to the weight of the deck.

PAUL BAUMANN,[10] M. AM. SOC. C. E. (by letter)[10a].—The deductions in this paper are based on the premise of a triangular distribution of bearing pressure on the haunches. All mathematical operations, based on this premise, lead to correct results, provided the premise is correct. The exact solution of the bearing-pressure problem on yielding supports is no simple one and leads to terms that would be entirely too complicated for the purpose of showing economy features of water-bearing members of buttressed dams.

The use of a simple, geometrical shape of the bearing pressure area, therefore, is justified, provided this shape—in this case a triangle—is a close approximation of the actual shape. If it is a close approximation, then the yield of the haunch, due to p at any distance, x, from a Y-axis through the end of the face slab (Fig. 4(b)) must be substantially equal to the deflection of the face slab due to p and W.

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[10] Chf. Designer, Quinton, Code & Hill – Leeds & Barnard Engrs. Consolidated, Los Angeles, Calif..

[10a] Received by the Secretary December 16, 1933.

For the elastic line of the face slab between $X = 0$ and $X = a$, due to bending:

$$E_1 I \frac{d^2y}{dx^2} = - M \quad \text{.......................(98)}$$

and for $M = \frac{p\,x^3}{6} - \frac{W\,x^2}{2}$:

$$E_1 I \frac{d^2y}{dx^2} = - \frac{1}{2} \left\{ \frac{p_0\,x^4}{3\,a} - W\,x^2 \right\} \quad \text{.................(99)}$$

in which, $\frac{p_0\,x}{a} = p$. Integrating Equation (99):

$$E_1 I \frac{dy}{dk} = - \frac{1}{2} \left\{ \frac{p_0\,x^5}{15\,a} - \frac{W x^3}{3} \right\}_0^a + C_1 \quad \text{.............(100)}$$

For $x = a$, the slope, $\frac{dy}{dk}$, may be determined by cutting the face slab in two at mid-span and by applying the stresses there as external forces the resultant of which is the maximum moment; thus:

$$M_0 = \frac{W}{6} \left[L_1 (s\,a + 3\,L_1) - a^2 \right] \quad \text{...............(101)}$$

which differs from the author's Equations (37) and (46), because the latter are based on a uniform distribution of bearing pressure which is in contradiction to the basic assumption of a triangular distribution.

The moment at any section between $x = a$ and $x = L_1 + a$, is:

$$M = M_0 - \frac{W\,(L_1 + a - x)^2}{2} \quad \text{................(102)}$$

Therefore,

$$E_1 I \frac{d^2y}{dx^2} = \left\{ M_0 - \frac{W}{2} \left[(L_1 + a)^2 - 2\,(L_1 + a)\,x + x^2 \right] \right\}$$

and,

$$E_1 I \frac{dy}{dx} = \left\{ M_0\,x - \frac{W}{2} \left[(L_1 + a)^2\,x - (L_1 + a)\,x^2 + \frac{x^3}{3} \right] \right\}_a^{L_1+a} + C \text{..(103)}$$

The constant, C, follows from the condition that $\frac{dy}{dx}$ at mid span must be zero; thus:

$$C = (L_1 - a) \left[\frac{W}{6}\,(L_1 + a)^2 - M_0 \right] \quad \text{..............(104)}$$

and for $x = a$:

$$E_1 I \frac{dy}{dx} = - \left\{ a \left[M_0 - \frac{W}{2} \left((L_1 + a)^2 - (L_1 + a)\,a + \frac{a^2}{3} \right) \right] \right.$$

$$\left. + (L_1 + a) \left[\frac{W}{6}\,(L_1 + a)^2 - M_0 \right] \right\} \quad \text{..............(105)}$$

Introducing Equation (105) in Equation (100):

$$C_1 = \frac{1}{E_1 I}\left\{a\left[M_0 - \frac{W}{2}\left((L_1 + a)^2 - (L_1 + a)\,a + \frac{a^2}{3}\right)\right.\right.$$
$$\left.\left. + (L_1 + a)\left[\frac{W}{6}(L_1 + a)^2 - M_0\right] + \frac{1}{2}\left[\frac{p_0\,a^4}{15} - \frac{W a^3}{3}\right]\right]\right\} \ldots\ldots (106)$$

Integrating again, the resulting simplified formula is the equation of the elastic line of the face slab due to bending; thus:

$$E_1 I y = -\frac{x^4}{12}\left\{\frac{p_0\,x^2}{15\,a} - \frac{W}{2}\right\} + x\left\{\frac{W}{6}\left[L_1^3 - a^3 + 12\,L_1\,a^2\right] - M_0\,L_1 + \frac{p_0\,a^4}{30}\right\} \ldots (107)$$

The deflection, $\Delta\, y_s$, due to shear of the face slab is:

$$\Delta\, y_s = \frac{1}{j_1\,G_1\,d_2}\left\{\frac{p_1\,x^4}{8\,a} - \frac{W\,x^2}{2}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (108)$$

Adding Equation (108) to the deflection due to bending:

$$E_1\,I\,(y + \Delta\, y_s) = E_1\,I\,y_F = -\frac{x^4}{12}\left\{\frac{p_0\,x^2}{15\,a} - \frac{W}{2}\right\} + x\left\{\frac{W}{6}(L_1^3 - a^3 + 12\,L_1\,a^2)\right.$$
$$\left. - M_0 L_1 + \frac{p_0\,a^4}{30}\right\} + \frac{1}{0.30\,j_1\,d_2}\left\{\frac{p_0\,x^4}{8\,a} - \frac{W\,x^2}{2}\right\} \ldots\ldots\ldots (109)$$

Neglecting the influence of bending as suggested by the author, the elastic yield of the haunch due to shear, Q, is:

$$y_H = -\frac{1}{j_2\,G_2}\int_0^a \frac{Q\,Q_A}{d}\,dx \ldots\ldots\ldots\ldots\ldots\ldots (110)$$

in which, $Q = \dfrac{p_0\,(a^2 - x^2)}{2\,a}$; $Q_A = 1$; and, $d = d_1 + S\,(a - x)$. Introducing these values in Equation (110):

$$y = \frac{-\,p_0}{2\,a j_2\,G_2}\int_0^a \frac{a^2 - x^2}{d_1 + s\,(a - x)}\,dx = -\,K\int_0^a \frac{a^2 - x^2}{d_1 + s\,(a - x)}\,dx$$
$$= -\,K\left\{\int \frac{a^2}{d_1 = s\,(a - x)}\,dx - \int \frac{x^2}{d + s\,(a - x)}\,dx\right\} \ldots\ldots (111)$$

Considering each integral separately:

$$\int \frac{a^2}{d_1 + s\,(a - x)}\,dx = -\frac{1}{s}\,l\,n\left\{d_1 + s\,(a - x)\right\} \ldots\ldots\ldots (112)$$

and, if v is substituted for $a - x$:

$$\int \frac{x^2}{d_1 + s\,(a - x)}\,dx = a^2\int \frac{-\,dv}{d_1 + sv} - 2a\int \frac{-v\,dv}{d_1 + sv} + \int \frac{v^2\,dv}{d_1 + sv} \ldots (113)$$

Then Equation (111) may be written:

$$y = -\,K\left[-\frac{2\,a}{s^2}\left\{d_1 + s\,(a - x) - d_1\,l\,n\left[d_1 + s\,(a - x)\right]\right\}\right.$$
$$+ \frac{1}{s^3}\left\{\frac{1}{2}\left[d_1 + s\,(a - x)\right]^2 - 2\,d_1\left[d_1 + s\,(a - x)\right]\right.$$
$$\left.\left. + d^2{}_1\,l\,n\left[d_1 + s\,(a - x)\right]\right]\right\} + C \ldots\ldots\ldots\ldots\ldots\ldots (114)$$

For $x = 0$, $y = 0$, and:

$$C = \frac{2\,a}{s^2}\left\{d_1 + s\,a - d_1\,l\,n\left[d_1 + s\,a\right]\right\} - \frac{1}{s^3}\left\{\frac{1}{2}\left[d_1 + s\,a\right]^2 - 2\,d_1\left[d_1 + s\,a\right] + d_1^2\,l\,n\left[d_1 + s\,a\right]\right\} \quad \text{............(115)}$$

After reducing and re-arranging:

$$y_H = \frac{-\,p_0}{2\,a\,j_2\,G_2\,s}\left\{\frac{d_1}{s}\left[l\,n\,\frac{d_1 + s\,(s - x)}{d_1 + s\,a}\right]\left(\frac{d_1}{s} + 2\,a\right) + \left[\frac{x^2}{2} - a\,x + \frac{d_1\,x}{s}\right]\right\} \quad \text{..................(116)}$$

which obviously does not conform to Equation 109.

A numerical example will serve to check this. In Fig. 3(a), let $H = 100$ ft; $W = 43.40$ lb per sq in.; $S = 3$; $p_0 = 700$ lb per sq in.; $\tau = 60$ lb per sq in.; and, $L_2 = 240$ in. Furthermore, let $A = \frac{d_1}{a} = 4$ and $b = 0$; then, from Equation (18), $a = \frac{2\,W\,L_2}{p_0} = 29.80$ in., and, therefore, $d_1 = 4 \times 29.80 = 119.20$ in. With these values, Equation (109) gives, for $x = a$:

$$E_1\,y_F = \left\{\frac{3\,400}{I} + \frac{8.70}{d_2}\right\}10^6$$

With sufficient approximation:

$$I = \frac{(d_2 + e_2)^3}{12} = \frac{(0.0625\,\sqrt{M_0} + 2.5)^3}{12} = 24\,200 \text{ in.}^4$$

in which, $0.0625\,\sqrt{M_0} = d_2 = 63.70$ in. Finally, $E_1\,y_F = 0.278 \times 10^6$ lb per in.

Introducing the same values in Equation (116) with $x = a$ and $G_2 = G_1 = 0.30\,E_1$:

$$E_1\,y_H = \frac{-\,700}{0.60 \times 29.80 \times 0.875 \times 3}\left\{-\,9\,509\right\} = 0.1415 \times 10^6 \text{ lb per in.}$$

which is roughly one-half as great as $E_1\,y_F$. The deflection of the haunch due to bending is of the order of magnitude of 0.002×10^6 and, therefore, would not appreciably adjust the discrepancy.

The distribution of the bearing pressure, p, on the haunch could readily be ascertained by means of photo-elastic tests on models for various, relative rigidities of haunch and face slab, and new equations for the configuration of the haunch could be derived based on the results thereof, which would establish compatibility between the two members of the structure.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ANALYSIS OF UNSYMMETRICAL CONCRETE ARCHES

Discussion

BY DAVID A. MOLITOR, M. AM. SOC. C. E.

DAVID A. MOLITOR,[16] M. AM. SOC. C. E. (by letter)[16a].—In this paper as in the previous one on design of symmetrical concrete arches,[2] the author uses the method of elastic weights for determining the influence lines for the

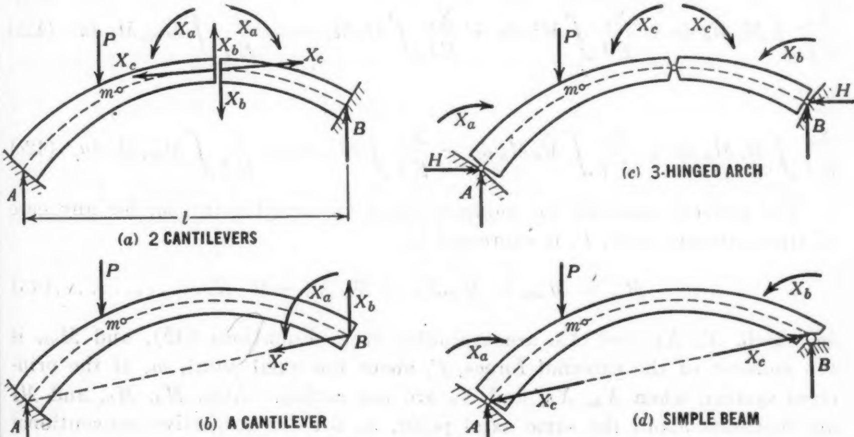

FIG. 14.—FIXED ARCHES. FOUR DIFFERENT PRINCIPAL SYSTEMS, EACH WITH THREE REDUNDANTS

reactions, and the problem of the unsymmetrical arch is solved by treating two halves of two different symmetrical arches.

Fixed Arches.—The problem of fixed arches involves three redundant conditions and there are at least four possible different assumptions as to choice

NOTE.—The paper by Charles S. Whitney, M. Am. Soc. C. E., was published in October, 1933, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[16] U. S. Constr. Engr., for Superv. Archt., U. S. Treasury Dept., Lansing, Mich.

[16a] Received by the Secretary December 11, 1933.

[2] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

of a principal system. These four cases are illustrated in Fig. 14, for which the redundant conditions are, as follows:

Fig. 14(*a*).—Moment, X_a; shear, X_b; and thrust, X_c.
Fig. 14(*b*).—Moment, X_a; shear, X_b; and thrust, X_c.
Fig. 14(*c*).—Moment, X_a; moment, X_b; and moment, X_c.
Fig. 14(*d*).—Moment, X_a; moment, X_b; and thrust, X_c.

The redundant conditions, X_a, X_b, X_c, represent forces or moments, respectively, as shown, and are externally applied to the principal system in each case. The three redundants are additional to the reactions produced by the actual dead and live loads, *P*, and all together, the redundants, reactions, and loads, must form a system in equilibrium, when applied to any one of the four principal systems as shown.

Neglecting the effect of axial thrust, the three redundants may be evaluated from three elasticity equations, expressing the relations between the external loads, *P*, and the redundants, *X*, as follows:

$$\frac{X_a}{E\,I}\int M^2_a\,du + \frac{X_b}{E\,I}\int M_a\,M_b\,du + \frac{X_c}{E\,I}\int M_a\,M_c\,du = \frac{1}{E\,I}\int M_{om}M_a\,du \quad (42a)$$

$$\frac{X_a}{E\,L}\int M_b M_a\,du + \frac{X_b}{E\,I}\int M^2_b\,du + \frac{X_c}{E\,L}\int M_b M_c\,du = \frac{1}{E\,I}\int M_{om}M_b\,du \quad (42b)$$

and,

$$\frac{X_a}{E\,I}\int M_c M_a\,du + \frac{X_b}{E\,I}\int M_c M_b\,du + \frac{X_c}{E\,I}\int M^2_c\,du = \frac{1}{E\,I}\int M_{om}M_c\,du \quad (42c)$$

The general equation for moment about any axial point, *m*, for any case of simultaneous loads, *P*, is expressed by:

$$M_m = M_{om} - M_a\,X_a - M_b\,X_b - M_c\,X_c \quad (43)$$

in which, X_a X_b, and X_c, are evaluated from Equations (42), and M_{om} is the moment of the external forces, *P*, about the axial point, *m*, of the principal system, when X_a, X_b, and X_c are not acting. Also, M_a, M_b, and M_c are moments about the same axial point, *m*, for the respective conventional loads, $X_a = 1$, $X_b = 1$, and $X_c = 1$, acting separately on the principal system, when the loads, *P*, are absent. Hence, all quantities in Equations (42) and (43), except the three *X*'s, are statically determinate by any of the four methods illustrated, and the three elasticity Equations (42), finally determine these redundants, X_a, X_b, X_c.

The author attempts the general solution of fixed unsymmetrical arches on the assumptions in Fig. 14(*a*), and immediately encounters difficulties because of the interruption in his principal system at the crown of the arch. These difficulties are entirely avoided when the principal system in Fig. 14(*d*) is chosen, since all equations then apply over the entire span, instead of suffering the discontinuity between the two cantilevers as in Fig. 14(*a*).

Simplifications.—The simplifications possible in dealing with the principal system (Fig. 14(d)), are illustrated in Fig. 15. Referring to Fig. 15(a), the redundant conditions are moment, X_a; shear, X_b; and thrust, X_c, applied to

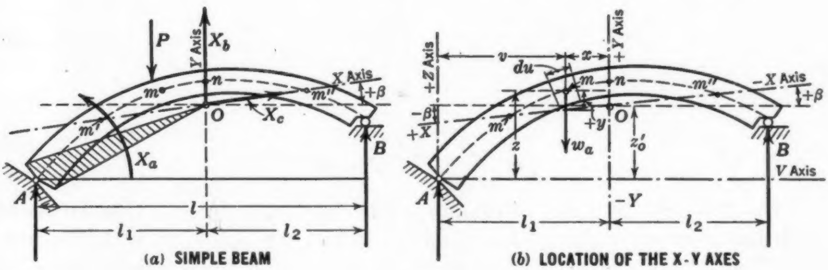

FIG. 15.

the rigid disk, AO. The point, O, is the center of gravity of all the elastic weights, $w_a = \dfrac{du}{E\,I}$. The redundants (Fig. 15(a)) are shifted to a point, O, such that only those terms in Equations (42), involving M_{om}, M^2_a, M^2_b, and M^2_c remain. All other terms become zero, thus avoiding the necessity of solving the three elasticity Equations (42), for simultaneous values of the X's. By introducing the elastic weights, $w_a = \dfrac{du}{E\,I}$, $w_b = x w_a$, and $w_c = y w_a$, into the remaining terms of Equations (42) and noting that $M_a = 1$, $M_b = x$, and $M_c = y \cos \beta$, as defined previously herein, the following simple relations are obtained after some reduction:

$$X_a = \frac{\Sigma\, M_{om}\, w_a}{\Sigma\, w_a} \qquad \text{(44a)}$$

$$X_b = \frac{\Sigma\, M_{om}\, w_b}{\Sigma\, x\, w_b} \qquad \text{(44b)}$$

and,

$$X_c = \frac{\Sigma\, M_{om}\, w_c}{\cos \beta\, \Sigma\, y\, w_c} \qquad \text{(44c)}$$

in which the integrations have been replaced by summations.

Fig. 15(a) shows the new disposition of the redundants when chosen so as to simplify the problem and avoid the solution of the three elasticity equations, while yet retaining the simple beam, $A\ B$, for a principal system.

The redundants are now applied to a rigid disk, $A\ O$, attached to the left end of the beam. A similar set of redundants, X, of equal and opposite kind (but not shown), is assumed acting at the right end of the beam.

The point, O, which is the center of gravity of the elastic weights, w_a, is located by taking moments of all the weights about two assumed axes, as

Z, V, with origin at A, to find the co-ordinates, l_1 and z'_o, as follows:

$$l_1 = \frac{\Sigma\, v\, w_a}{\Sigma\, w_a} \quad \text{.........................(45)}$$

$$z'_0 = \frac{\Sigma\, z\, w_a}{\Sigma\, w_a} \quad \text{.........................(46)}$$

and,

$$\tan \beta = - \frac{\Sigma\, x\, z\, w_a}{\Sigma\, x^2\, w_a} \quad \text{.....................(47)}$$

The new axes of the co-ordinates, with the origin, O, may now be drawn with the Y-axis vertical and the X-axis making an angle, β, with the horizontal, such that the centrifugal moment, $\Sigma\, x\, y\, w_a$, of the elastic weights, w_a, about O must be zero. (See Fig. 15(b)). For symmetrical arches, $l_1 = l_2 = \frac{1}{2}\, l$, and $\beta = O$; and, the ordinate, $z'_0 = \frac{\Sigma\, z\, w_a}{\Sigma\, w_a}$, alone locates the co-ordinate axes, X, Y, with the origin at O.

With the redundants chosen so that X_b is the vertical component along the Y-axis, X_c, the thrust along the X-axis, and X_a, a moment applied to the disk, $A\, O$, then Equation (43) may be reduced to its simplest terms.

Referring to Fig. 15(b), the following values are apparent from the foregoing definitions: $M_a = 1$ is the moment about any axial point, m, produced by the moment, $X_a = 1$, applied to the principal system; $M_b = 1\, x$ is the moment about this same point, m, due to the force, $X_b = 1$, acting on the principal system; and, $M_c = 1\, y \cos \beta$ is the moment about this same point, m, due to the force, $X_c = 1$, acting on the principal system. These values introduced into Equation (43) give the general moment equation for any axial point, m, of an unsymmetrical arch, as:

$$M_m = M_{om} - 1\, X_a - x_m\, X_b - y_m \cos \beta\, X_c \quad \text{.........(48)}$$

in which, M_{om} is the moment about the axial point, m, due to the actual loads, P, acting on the principal system. The point, m, has the co-ordinates, x_m, y_m. When the arch is symmetrical the angle, β, becomes zero and $\cos \beta = 1$.

The foregoing outline thus affords a complete solution of the geometry of all fixed arch problems by one and the same set of equations, and following the same method of analysis, except that the unsymmetrical case involves more labor.

Critical Stresses.—For the unsymmetrical arch critical stresses will occur at the five sections, A, m', n, m'', and at B, as may be shown by discussing the values for M_m as given by Equation (48) for these five sections.

For a symmetrical arch only three sections, A, m', and n, involve critical stresses. The critical stresses represent the maxima of all possible maximum stresses, thus rendering unnecessary the stress analysis for any arch sections other than those herein designated.

Importance of Shape of Axial Line as Affecting Economic Design.—The most important single factor in economic arch design is the shape of the arch axis, which should be a three-centered or five-centered curve as nearly as possible coincident with the line of thrust drawn for total dead load plus one-half the equivalent uniform live load over the entire span. The dead-load pressure line advocated by the author, and approximated by a transformed catenary, will not result in equal positive and negative moments for the critical sections, and is, therefore, not the shape for maximum economy.

Concluding Remarks.—The writer has aimed to emphasize the importance of the fundamental assumptions which lead to the most acceptable solution of the general problem of fixed arches whether symmetrical or otherwise.

The error of the author in starting out with a solution for symmetrical arches which is well enough as far as it goes, certainly leads to unnecessary complications when adapted to the unsymmetrical form. It is far better to solve the geometry in the simplest possible manner for the unsymmetrical case and then introduce the slight simplifications occasioned by the special case of symmetry, as the writer has done in this discussion. In other words, the solution herein indicated is applicable to any arch problem, the only difference being in the extra numerical work which is always involved in solving the unsymmetrical case.

The author claims: (*a*) That the simplicity of his method of analysis permits a more thorough study of the proportions of an unsymmetrical arch; (*b*) that this should result in a more satisfactory design; and (*c*) that an attempt is made to clarify the action of the unsymmetrical arch so that each step can be readily understood by the designer. The author has apparently failed to substantiate these claims.

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DISCUSSIONS

DEFORMATION OF STEEL REINFORCEMENT DURING AND AFTER CONSTRUCTION

Discussion

BY F. N. MENEFEE, M. AM. SOC. C. E.

F. N. MENEFEE,[17] M. AM. SOC. C. E. (by letter)[17a].—Shrinkage cracks in concrete vary from the minute "crazing" cracks seen only by the aid of a magnifying glass (and which are as shallow as they are minute) to the large cracks one sees in floors, retaining walls, beams, girders, and columns where no provision has been made to prevent their forming and which may extend completely across the member. Since these latter cracks were preceded by tension in the member, in most cases induced by shrinkage, and since good concrete under favorable conditions may stand a tension of 400 lb per sq in. for a short time, it should be obvious that considerable concrete is in tension, varying from small insignificant amounts to amounts just below failure, after which the tension in the concrete at the point of failure is relieved.

Laboratory tests on mortars were probably first made by Considére, who found 1:3 plain mortars shrunk from 0.05% to 0.15% when hardened in air. Neat cements shrink more, as was to be expected, since the sand resists shrinkage and the compression induced by shrinkage, and the volume of water is greater. These values have been checked many times since, the actual values depending on the amount of cement, the quantity of water, and the nature of the matrix. In general, with all other conditions the same, rate of evaporation of the water determines the elapsed time before the maximum deformation is reached. In the test recorded in Fig. 9 the forms were of steel; hence the rate of shrinkage was relatively slow.

In a general way the outside limits of the compression in the steel due to shrinkage of the concrete may be arrived at by considering the tensile strength of the concrete, which seldom exceeds 400 lb per sq in. ultimate. In the columns investigated by the author the gross area of the concrete base-

NOTE.—The paper by Sergius I. Sergev, Esq., was published in October, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. William Russell Davis and J. R. Worcester; and January, 1934, by Messrs. F. E. Richart and Homer M. Hadley.

[17] Prof., Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

[17a] Received by the Secretary December 26, 1933.

ment column was 325 sq in. If, due to shrinkage, the full value of 400 lb per sq in. was developed in the concrete, the total tension would amount to 130 000 lb, which must necessarily be resisted by compression in the steel. With 6.2 sq in. of steel, this amounts to 21 000 lb per sq in., which is approximately the value observed by the author on the basement columns at the end of 800 days, after the computed stress due to load is subtracted.

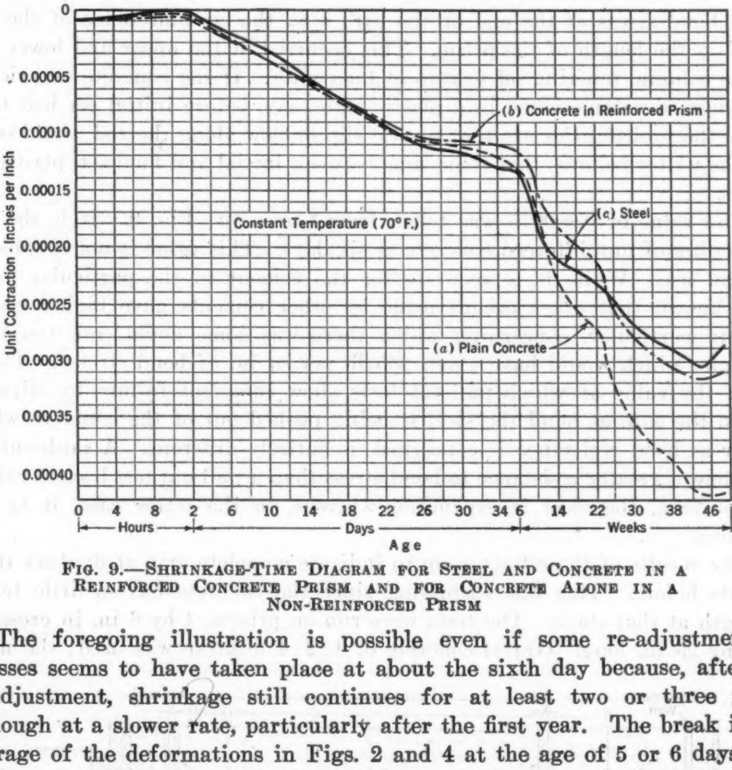

FIG. 9.—SHRINKAGE-TIME DIAGRAM FOR STEEL AND CONCRETE IN A REINFORCED CONCRETE PRISM AND FOR CONCRETE ALONE IN A NON-REINFORCED PRISM

The foregoing illustration is possible even if some re-adjustment of stresses seems to have taken place at about the sixth day because, after the re-adjustment, shrinkage still continues for at least two or three years, although at a slower rate, particularly after the first year. The break in the average of the deformations in Figs. 2 and 4 at the age of 5 or 6 days may be due to re-adjustments or minor failures, as the author indicates, because work on the structure may have set up vibrations which caused the shrinking concrete to fail in tension. At this age ordinary 5 000-lb concrete would not stand much more than 250 lb per sq in. of steadily applied tension. According to Fig. 2 the unit compressive stress in the steel was 10 500 lb per sq in. (amounting to a total of 65 000 lb), which, in turn, would be the amount of tension in 325 sq in. of concrete, or 200 lb per sq in. This stress on 6-day concrete, with vibrations incident to pouring a floor over it, might easily cause tension failure in the concrete with a consequent relief of the compression in the steel.

From the results of his own tests, designed to determine when gripping of the concrete is complete, the writer believes that it is not likely that there was a failure of bond at six days, because the maximum bond stress

was only 26.5 lb per sq in., using 0.000015 in. per in. as the shortening of plain concrete in the formula[18]:

$$P_y = \frac{uol}{4}\left[1 - \left(\frac{2\,y}{l}\right)^2\right] \dots\dots\dots\dots\dots\dots (11)$$

in which, P_y = the total compressive stress in the bar = the total tensile stress in concrete, at any section, y, distant from the center; u = the maximum bond stress at the end of the bar; o = the circumference of the bar; and l = the length of the prism. This occurred at the upper and lower ends of the column, tapering off to zero at the center. If the concrete experienced a change in volume when in a plastic state, say, before initial set had taken place, the probabilities are that plastic slip or flow along the rod would occur, but the writer believes that some time between initial and final set, plastic slip ceases.

The value of 0.000015 was taken from Curve (*a*) Fig. 9. It is the unit shortening of unreinforced concrete at six days. This value is not reliable for general use. While it is accurate for the cement in the particular prism, it is known that other prisms made by other cements gave higher results. One in particular, concerning which there was some doubt, ran ten times as high (which would have given 265 lb per sq in. of bond stress). This is about the value at which pull-out tests show incipient failure by slipping; but in the case at hand the steel is being pushed out of the concrete which, as far as bond resistance is concerned, is entirely different. A push-out test will show a greater resistance to bond stress than a pull-out test because, in the former case, the steel is expanding whereas, in the latter case, it is contracting.

The results of these tests seem to indicate complete grip at final set if not a little before. This was surprising since the concrete has so little tensile strength at that stage. The tests were run on prisms, 4 by 6 in. in cross-section by 29 in. long. Gravel concrete of 1 : 2 : 4 mixture was used; the maxi-

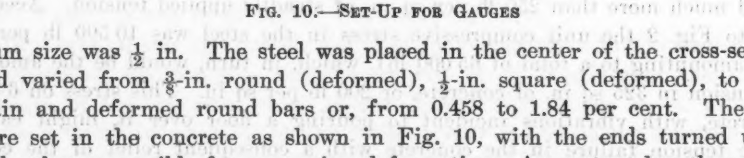

FIG. 10.—SET-UP FOR GAUGES

mum size was $\frac{1}{2}$ in. The steel was placed in the center of the cross-section and varied from $\frac{3}{8}$-in. round (deformed), $\frac{1}{2}$-in. square (deformed), to $\frac{3}{4}$-in. plain and deformed round bars or, from 0.458 to 1.84 per cent. The bars were set in the concrete as shown in Fig. 10, with the ends turned up to make them accessible for measuring deformation. A constant length-measuring reference gauge was used, in which any change in length of one part, due to temperature, was offset by an equal change in the opposite direction

[18] *Bulletin No. 126*, Eng. Experiment Station, Univ. of Illinois, Equation (1), p. 20.

by another part (see Fig. 11). A roller with an extension arm and mirror was placed between the brass bearing-block and the material to be measured. Readings were made by means of a telescope and a scale set at a distance in relation to the radius of the knife-edged roller, so that $\frac{1}{32}$ in. on the scale meant a change in length of the steel or concrete of 0.00001 in. The constancy of the length of the metal frame resting on the roller was determined by means of water baths in which the temperature could be controlled. Any change in length of the reinforcement steel or concrete produced a rotation of the mirror and a change in the scale reading as viewed through the telescope. Readings were begun 15 min after the water was placed on the con-

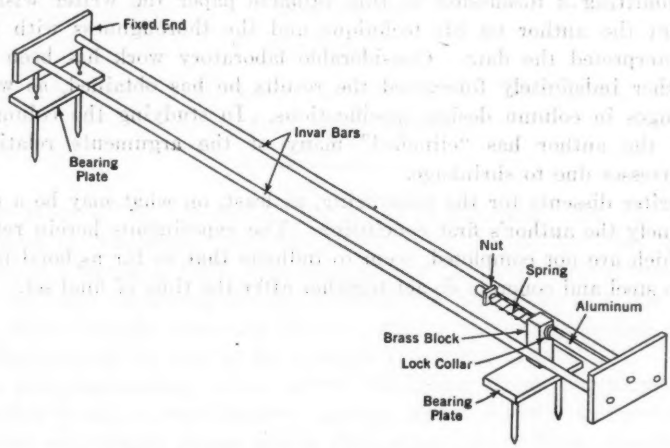

FIG. 11.—CONSTANT LENGTH REFERENCE GAUGE

crete (which, previously, had been thoroughly dry-mixed) and were continued every few minutes throughout the first few hours. Then they were reduced in frequency as time went on until the end of seven days, and, finally, were taken once every two weeks.

In general, the results of this investigation indicated that by the end of the second hour the steel and the concrete were shrinking at the same rate. For some unexplained reason the steel seemed to shrink faster during the first two hours than the concrete. For some time this led the writer to believe that there was something wrong with the reference gauge or the measuring devices; however, with no change or interference on the part of the observers, the shrinkage curves of the concrete and steel generally ran very close to parallel after the end of the second hour.

Fig. 9 is a 48-week record of two prisms exactly alike, except that one (Curve (*b*)) is reinforced and the other (Curve (*a*)) is not. It will be noted that the plain concrete prism as well as the reinforced one increased in length for about a day, after which shrinkage began; the length passed from positive to negative on the third day, after which the shrinkage continued at about the same rate. Not all the prisms tested showed an increase in length

in the early stages of their history. In some cases the shrinkage was noticed at the very beginning. For some reason which still remains unexplainable by the writer, the steel generally shortened at a greater rate during the first two hours, after which the shrinkage curves for steel and concrete were substantially parallel.

In other cases where the concrete increased in volume the reaction began just before, or at the time of, final set, and shrinkage set in. Of twelve prisms, all but two showed practical parallelism of the change-in-length curves for steel and the concrete after the second hour. It seems, however, that this might depend somewhat on the percentage of steel.

In submitting a discussion to this valuable paper the writer wishes to compliment the author on his technique and the thoroughness with which he has interpreted the data. Considerable laboratory work has been done, which rather indefinitely forecasted the results he has obtained, as well as some changes in column design specifications. In studying the columns in the field, the author has "clinched" many of the arguments relative to induced stresses due to shrinkage.

The writer dissents for the time being, at least, on what may be a minor point, namely the author's first conclusion. The experiments herein referred to, but which are not completed, seem to indicate that as far as bond is concerned the steel and concrete do act together after the time of final set.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

INTERCEPTING SEWERS AND STORM STAND-BY TANKS AT COLUMBUS, OHIO

Discussion

BY MESSRS. C. B. HOOVER AND C. D. MCGUIRE, AND JULIAN MONTGOMERY

C. B. HOOVER,[11] M. AM. SOC. C. E., and C. D. MCGUIRE,[12] ESQ. (by letter)[12a]. —Previous to November, 1932, the drainage of the Alum Creek Sewerage District, in Columbus, Ohio, was pumped over the Alum Creek-Scioto River divide into the Olentangy-Scioto Intercepting Sewer District for delivery to the Main Sewage Pumping Station, which, in turn, delivered this and other drainage of the city to the Sewage Treatment Works.

The pumping station, which served the Alum Creek Drainage District, was equipped with a sand-catcher, tandem coarse screens, and motor-driven horizontal centrifugal pumps which discharged into a 20-in. cast-iron force main, 8 200 ft in length. This station was in operation for twenty-three years previous to the completion of the storm stand-by tanks, during which time about 14 300 000 000 gal of sewage were pumped over the divide. Very little, if any, of this pumpage included storm drainage, and there were many interruptions of service due to floods in the creek and a lack of operating funds.

The minimum flow in Alum Creek during the warm weather season is about 0.5 cu ft per sec per 1 000 persons tributary to the district. Consequently, the wastage of sewage during storm flows constituted a nuisance menace both in theory and in fact.

Inspections of the creek below the main outfall sewer and the pumping station during the past five years indicate the condition of the water of the stream during 1929, 1930, 1931, and 1932, before the storm stand-by tanks and cross-town sewer were placed in operation, and during 1933, when the entire flow of the District was handled by this gravity arrangement. The bio-chemical oxygen demand values of the stream water during the summer

NOTE.—The paper by John H. Gregory, R. H. Simpson, Orris Bonney, and Robert A. Allton, Members, Am. Soc. C. E., was published in October, 1933, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

[11] Supt., Divs. of Water and Sewage Disposal, Columbus, Ohio.

[12] Chemist in Chg., Div. of Sewage Disposal, Columbus, Ohio

[12a] Received by the Secretary December 11, 1933.

seasons of the years indicated and, as shown in Table 7(*a*), reveal the improvement accomplished by this change in the method of delivering this drainage to its intended outlet.

An inspection of Table 7(*a*) will show that the average 5-day bio-chemical oxygen demand of the stream water previous to 1933 was very much in excess

TABLE 7.—CONDITION OF ALUM CREEK BELOW MAIN OUTLET SEWER AT LIVINGSTON AVENUE, COLUMBUS, OHIO, BEFORE AND AFTER THE ALUM CREEK STORM STAND-BY TANKS WENT INTO OPERATION ON NOVEMBER 16, 1932

Month	1929	1930	1931	1932	1933	Dissolved oxygen in the creek water, in parts per million, in 1933
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(*a*) FIVE-DAY BIO-CHEMICAL OXYGEN DEMAND OF THE CREEK WATER, IN PARTS PER MILLION, AT 20° CENTIGRADE						
June	......	7.0	6.0	6.6	4.8	5.0
	......	18.0	17.0	14.0	1.1	4.7
	......	10.4	13.2	5.2	2.5	5.3
	......	14.4	13.0	......	2.8	3.7
July	......	11.0	11.0	12.0	3.2	3.4
	......	32.0	10.4	22.0	2.4	5.0
	......	22.8	19.2	16.2	3.7	2.1
	11.0	28.8	7.2	7.2	2.8	5.7
	......	34.8	8.4	......	1.8	5.4
August	12.0	27.0	6.6	8.0	2.9	6.2
	12.5	39.2	5.6	5.0	2.1	5.6
	6.3	15.4	8.0	11.2	2.6	4.2
	10.0	......	......	4.8	......	...
	11.0	......	......	......	......	...
September	20.0	18.2	3.6	5.6	2.9	2.6
	14.0	33.6	8.4	34.2	6.4	4.8
	9.0	25.2	8.8	61.0	3.8	6.7
	6.5	26.6	28.0	19.5	1.6	7.1
	......	......	26.4	......	......	...
October	11.0	47.6	49.6	13.0	3.8	8.4
	23.0	34.8	33.6	36.0	2.2	7.2
	21.0	21.6	28.0	2.4	0.8	7.3
	12.6	23.8	12.0	10.5	......	...
	11.4	16.8	......	......	......	...
Sums	191.3	509.0	324.0	294.4	54.2	100.4
Means	12.8	24.2	15.4	15.5	2.9	5.3
(*b*) STREAM FLOW FOR THE PERIOD, JUNE TO OCTOBER, INCLUSIVE, IN CUBIC FEET PER SECOND						
Mean	67.8	9.1	62.0	24.9	15.2	...
Maximum	1240	42	1220	634	150	...
Minimum	7.3	3.4	2.4	2.5	3.5	...

of the dissolved oxygen saturation point of the water. In 1933, on the contrary, the oxygen balance was very much on the safe side; that is, there was ample dissolved oxygen in the water to satisfy the oxygen demand. The average summer-time oxygen demand of the stream water previous to 1933 was about 17 ppm, whereas, in 1933, when the mean flow of the creek was below the average for 1929 to 1932, inclusive, the oxygen demand was 2.9 ppm, or a reduction of 83% from that which obtained previously. The values

in Table 7(*b*) are from records of the U. S. Geological Survey for the gauge on Alum Creek, 0,25 mile south of Livingston Avenue (see Fig. 1). The data in Table 7 (*a*) were obtained from tests taken on weekly inspections of Alum Creek.

JULIAN MONTGOMERY,[13] M. AM. SOC. C. E. (by letter)[13a].—Before sewage treatment works were considered necessary to prevent the pollution of streams, combined sewerage systems were considered practical and economical. Since 1900 probably not more than 1% of the sewerage systems constructed, have been combined systems. To-day, practically all State Health Boards require sewerage systems to be designed on the separate plan.

It is interesting to note that sewers first were constructed at Columbus, Ohio, in 1853, eighty years ago, and that they are functioning very well to-day. It is not surprising, furthermore, to note that the present plans call for using separate systems wherever possible.

In an experience of twenty years the writer has never designed a combined system. On several occasions he has been forced, by reasons of sanitation and economics, to reduce grades below the minimum recommended. Ordinarily, it has been the custom to use $n = 0.015$ for Kutter's formula in designing sewers; whereas for the pipe manufactured during the last few years the use of $n = 0.013$ would more nearly approach the true conditions. It is the writer's experience that by using care in the inspection and laying of pipe, flatter grades than those usually recommended, can be used without inconvenience.

In designing sanitary sewerage systems at Austin and Wichita Falls, Tex., 125 gal per capita daily was the value assumed. In 1916, weir measurements of the sewage flow at Austin gave an average daily flow of 55 gal per capita contributing. In 1925, weir measurements of the sewage flow at Wichita Falls gave an average daily flow of 60 gal per capita contributing.

To-day, the writer assumes for the design of sewerage systems for cities with populations of less than 7 500 an average of 60 gal daily per capita contributing, which also includes infiltration. It should be noted that for cities in the semi-arid districts, exfiltration deserves more consideration than infiltration. The idea at all times, of course, is to construct water-tight joints.

The design of both storm and sanitary sewerage systems calls for various assumptions, all of which affect the size of the mains and laterals, and, accordingly, the cost of the system. Not always should the same assumptions be made for the small city that are made for the larger cities in the design of a sanitary sewerage system. If this is done, dead capital will be sometimes invested, and the resulting system will produce a septic effluent.

A study of a complete plan for the given city results in the making of logical design assumptions. When a city has been zoned into its residential, commercial, and industrial districts, when the main highway arteries have been laid out, and when the other utility systems have been designed logically

[13] Cons. Engr. (Montgomery & Ward), Wichita Falls, Tex.

[13a] Received by the Secretary December 26, 1933.

for a reasonable period into the future, then the more logically and accurately can the assumptions be made for designing the storm and sanitary sewerage systems.

The inclusion of storm stand-by tanks in the plans for the Columbus System presents a new field for thought and study. The authors evidently took every practical condition into consideration in the design. Their preliminary studies may form the nucleus for a future treatise on the subject. The cost of such structures, of course, must always be balanced against the benefits to be derived.

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DISCUSSIONS

SOME SOIL PRESSURE TESTS

Discussion

BY MESSRS. EUGENE E. HALMOS, AND L. C. WILCOXEN

EUGENE E. HALMOS,[4] M. AM. SOC. C. E. (by letter)[4a].—A large amount of work was done in gathering data for this paper, both in the laboratory and in the office, and the writer wishes to pay his compliments to Mr. Parsons' ingenuity for having been able to condense the voluminous information collected into such a short paper. All the essential findings of the tests and the author's analytical studies have been included in a manner which can be easily digested by the reader.

Tests to determine the pressure of earth on retaining walls have been in progress for nearly a century, the purpose of such tests being, first, to develop formulas for the design of retaining walls, and, later, when the theory of earth pressure was more fully developed, to determine the value of the constants entering into such formulas. Most of the previous tests were conducted with dry materials and, in many cases, the materials were selected to be truly granular so as to come more closely within the scope of cohesionless semifluids. The writer has no knowledge of any tests conducted along the lines described by Mr. Parsons and believes that these are the first in which the influence of irrigation and drainage was determined throughout the entire height of a fill consisting of "run-of-bank" material.

The writer agrees with Mr. Parsons that considerably more data will have to be collected and studied before certain peculiar characteristics disclosed by the tests can be fully explained. Such a peculiarity is the fall of the pressure at the first stages of the irrigation; another is the material decrease of that part of the pressure attributed to the earth alone as the irrigation approaches the full height of the fill. This latter may be due to "arching" or "bin action" transferring part of the weight of the fill to the side walls of the apparatus by means of friction. It is not clear why such action should

NOTE.—The paper by H. de B. Parsons, M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1934, by J. C. Meem, M. Am. Soc. C. E.

[4] Chf. Engr., Parklap Constr. Corporation, New York, N. Y.

[4a] Received by the Secretary December 15, 1933.

be accentuated under irrigated condition. Neglect of either or both of these phenomena, however, results in a conservative design.

The most interesting and the most important practical disclosure is that described in Conclusion (7) which states the fact that after full drainage the residual pressure was found to be much greater than that of the originally dry earth. This should have an appreciable bearing on the assumptions underlying the safe design of retaining walls in which the supported bank has no positive surface drainage and, therefore, it is conceivable that after heavy rains the earth fill may be in the same condition of wet consistency as that of the tested fill after drainage was completed.

Retaining walls are designed usually on the assumption that as long as weep-holes, about 3 in. in diameter and about 25 ft apart, are placed in the lower part of the wall, the earth pressure during and after rains will remain the same as that estimated for the dry condition of the back-fill. A substantial proportion of retaining wall failures have occurred after heavy storms, indicating that more knowledge is needed on the pressure exerted by a back-fill alternately wetted and drained. Mr. Parson's experiments show that this pressure is about twice the assumed thrust, when the draining is from a fully saturated state.

Further tests should be made, simulating the effect of heavy and extended rain, with weep-holes inserted in the bottom of the test wall, or with subdrains at the back of the wall, in order to determine how much additional pressure should be assumed in the design to compensate for the wet consistency of the back-fill resulting from any expected rate of rainfall.

It may be argued that the test results referred to in Conclusion (7) are brought forth by the "passive" resistance of the fill which prevents the bulkhead from moving toward the fill when the outward pressure is diminished. The writer believes that the effect of this resistance should be negligible, considering the extremely small movements of the bulkhead. However, if the tests are repeated, it is suggested that measurements be made to determine the vertical component of the earth pressure and the magnitude of the elastic deformation of both the bulkhead and the side walls of the apparatus.

L. C. Wilcoxen,[5] Assoc. M. Am. Soc. C. E. (by letter)[5a].—The total pressure of sand on a bulkhead is reduced when the lower 1-ft stratum is immersed. This is a paradox that the author demonstrates positively. In view of the fact that he does not give a satisfactory explanation of the cause, the writer will relate a simple test that gives the clue to the nature of the phenomenon. An analysis based thereon follows: A glass jar was partitioned vertically and on one side a dry mortar sand was rammed level with the top. Water was then poured slowly into the open side. It was found that the water in the sand rose and remained the full 6 in. above the level on the open side. The difference in levels was due to the capillary action of the sand. This is the cue to a physical action that takes place when sand behind a bulkhead is immersed.

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[5] Asst. Civ. Engr., City Engr.'s Office, Detroit, Mich.

[5a] Received by the Secretary December 15, 1933.

Consider the case of a hypothetical bulkhead, 1 ft wide and 6.903 ft high, backed by a sand weighing 112.5 lb per cu ft, having a void ratio of 30.48% (solid ratio, 69.52%), and a horizontal reaction when dry of 21.88 lb per sq ft per ft of depth; that is, the ratio of the horizontal to vertical reaction is 0.194. The reaction diagram of this sand is shown (Fig. 7 (*b*)) in comparison with a similar one for water of the same depth (Fig. 7 (*a*)).

If the water level is raised to any height on the bulkhead, capillary action will cause it to rise, say, 6 in. higher in the sand behind. Several results accrue from the introduction of the water: First, there is the buoyancy of the

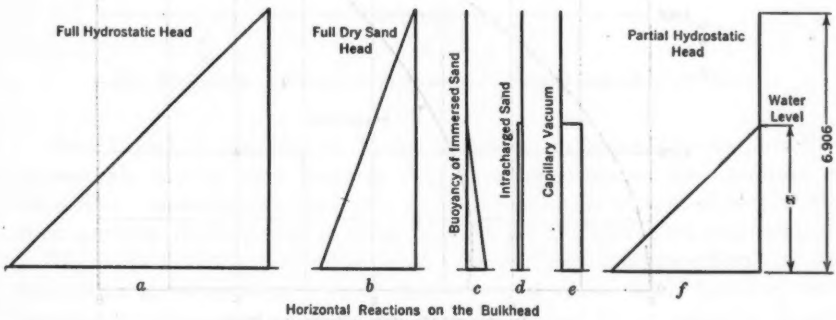

FIG. 7.

immersed sand with its proportionate reduction of the horizontal reaction; second, the sand is intracharged at the surface of the water, by the weight of water suspended by capillary action (this proportionally increases the horizontal reaction of the sand on the bulkhead); third, downward from the level of the water in the sand, the capillary action induces a vacuum pressure equal to the height of water supported by it; and, fourth, from the same level downward the hydrostatic head is effective.

Assuming a capillary head of 6 in. and the weight of water as 62.22, the quantitative results will be as follows: First, the buoyancy of the immersed sand will be ($0.6952 \times 62.22 =$) 43.25 lb per cu ft, and the resulting reduction in the horizontal reaction will be ($43.25 \times 0.194 =$) 8.39 lb per sq ft per ft of depth (Fig. 7 (*c*)); second, the intracharge due to the 6 in. of water suspended at its surface in the sand, will be ($0.5 \times 62.22 =$) 31.11 lb per sq ft, resulting in an increased horizontal sand reaction of ($31.11 \times 0.194 =$) 6.03 lb per sq ft (Fig. 7 (*d*)); third, the vacuum pressure due to capillary action will be ($0.5 \times 62.22 =$) 31.11 lb per sq ft (Fig. 7 (*e*)); and, fourth, the hydrostatic head will be 62.22 lb per sq ft of depth (Fig. 7 (*f*)).

From this it may be shown that the total horizontal reactions, in pounds, will be: $536 - 25.08x + 26.92x^2$, when x is the height of the water surface in the sand. This indicates that as the lower stratum of sand is immersed, the total pressure is decreased, the minimum pressure occurring when $x = 0.44$. At this point the decreased pressure is 5.8 lb. The computed pressure curves of water only and the irrigated sand for the author's bulkhead are

shown (Fig. 8). They check closely the experimental results of Fig. 3. At the full head, the author observes a difference between the total reactions of

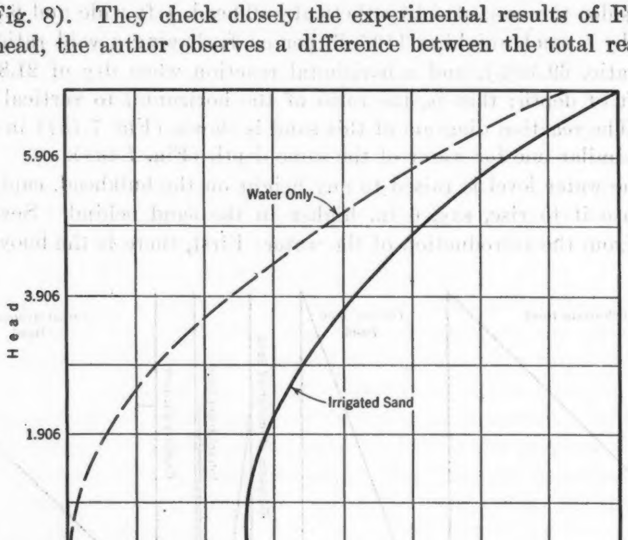

FIG. 8.

water and irrigated sand of (7 691 − 7 080 =) 611 lb, whereas the writer's computed difference from the equation is (8 041 − 7 309 =) 732 lb.

The author is to be complimented for the accuracy of his tests, which have brought to light so interesting a phenomenon.

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DISCUSSIONS

LINCOLN HIGHWAY FROM JERSEY CITY TO ELIZABETH, NEW JERSEY

Discussion

BY MESSRS. FRED LAVIS, AND THEODORE BELZNER

FRED LAVIS,[3] M. AM. SOC. C. E. (by letter)[3a].—So thoroughly has the author covered his subject that there is very little opportunity for comment or discussion. Nevertheless, emphasis may be placed on certain phases of the work, particularly in regard to those which affect engineers and engineering.

The author states that eight years elapsed from the "inception" of the work until its completion. These eight years of active work, however, were preceded by almost two years of preliminary studies[4] by an Advisory Board, under the Chairmanship of William G. Sloan, M. Am. Soc. C. E., the Highway Commission, and the State Legislature, and it seems worth while to emphasize the fact of this lapse of time and also of the almost incessant negotiations with all kinds of people and entities whose interests were affected throughout almost the entire period. The burden of these negotiations fell very largely on the Engineering Department of the State Highway Commission and on the counties and municipalities affected. The point to be emphasized is the importance of recognizing this as a part of engineering work and that this latter is by no means confined to computing the size of an I-beam or the proper mixture of concrete.

In the valuations of public utilities and other enterprises, due credit and allowance are seldom given for the time involved and the necessary costs of these factors; that is, first, the item of "Interest during Construction" and, second, the "Costs of Engineering."

It should be made quite clear, also, that there are two elements, or two major types, of design in a project of this nature: (1) The design of the route; and, (2) the design of the structure which carries the route. In both these designs the element of engineering expense is greatly increased by the need of meeting the numerous and varying demands of the many interested parties. It should not be thought that this engineering work and these de-

NOTE.—The paper by Sigvald Johannesson, M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[3] Cons. Engr., New York, N. Y.

[3a] Received by the Secretary December 13, 1933.

[4] Report of Advisory Board to the New Jersey State Highway Commission, August 8, 1923.

mands are unnecessary or unavoidable, but their cost and the time necessary to meet them should be recognized as an important element in a project of this nature.

The engineers in charge of this work visualized its general nature, importance, and effect from the very beginning, as is witnessed by the report of the Advisory Board; but naturally these factors were not so clear to the public and other interested entities. An infinitude of details also developed as soon as actual surveys and designs were started, and their effect on the studies continued throughout the course of the work.

The writer has previously mentioned[5] the various routes through Jersey City of which more or less detailed studies were made. This was also true of the route through Elizabeth. In the matter of structures, a number of schemes, including a viaduct of reinforced concrete arches of varying length of span was worked out for the meadows crossing where now the high viaduct has been built. Fairly detailed studies were made for lift spans for the river crossings, one of which would have been, if built, the heaviest lift span in existence, and these are only instances of the many phases of the studies.

It is generally true, as the author points out, that there is nothing particularly novel or unusual in any of the types of structures finally adopted; but the fact should not be lost sight of that the project as a whole was new and unusual and that it required not only great patience and not a little ingenuity in the negotiations. All these things required the expenditure of much time and considerable money.

It has been the writer's experience, and that of many engineers, that all this work of preparing sketches, plans, detailed studies, innumerable estimates of costs, etc., for various schemes eventually to be discarded, is apt to be entirely forgotten after the structure is completed; or, it is assumed that the engineers should have brought forth, full fledged, the final design at the beginning.

Even in private enterprises, where there may be less diversity of opinion, this is not possible or practical. It is much less so in public enterprises where State, county, and municipal officials, Chambers of Commerce, other public bodies, and the public itself, all have ideas, often many different ideas, to all of which some attention has to be paid. It is quite difficult to obtain recognition by public officials of this phase of engineering, and the work involved in the development of these projects, and it seems important, therefore, to record this and to give due recognition to the fact that the State Highway Commission of New Jersey and the State Highway Engineer never failed to give those in charge of the project complete support and full recognition of the work required, and which was accomplished, in working out the problems of this important enterprise.

The writer would like to express his own personal appreciation of the work of the author for the skill and ingenuity which he displayed in working out so many solutions of the problems presented, which are not referred to in the really modest record of his work which is set forth in this paper.

[5] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

THEODORE BELZNER,[6] AFFILIATE, AM. SOC. C. E. (by letter)[6a].—The author has covered his subject so thoroughly that discussion is limited to a few remarks on the subject of expansion joints.

The writer has had occasion to observe the behavior of the expansion joints shown in Fig. 9, and of many other types, all of them giving similar results, under heavy vehicular traffic.

Of them all, the writer believes firmly that Type 4, Fig. 9, is not only an ideal, efficient, and economical one from a maintenance point of view, but that it is the most serviceable type for heavy vehicular traffic on viaduct structures.

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[6] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Plant and Structures, City of New York, Brooklyn, N. Y.

[6a] Received by the Secretary January 2, 1934.

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DISCUSSIONS

PRACTICAL RIVER LABORATORY HYDRAULICS

Discussion

BY MESSRS LORENZ G. STRAUB, PAUL W. THOMPSON, RALPH W. POWELL, K. D. NICHOLS, AND FRANK W. EDWARDS

LORENZ G. STRAUB,[12] JUN. AM. SOC. C. E. (by letter)[12a].—The treatment given by Lieut. Vogel to practical river laboratory hydraulics is timely and especially interesting because of the author's connection with the development of the U. S. Waterways Laboratory. He points out in a clear manner the value of the laboratory in solving a variety of river-improvement problems.

The writer has used a nomenclature similar to that proposed by Lieut. Vogel and finds it especially convenient. Most nomenclatures used heretofore in presenting the principles of mechanical similitude are complicated with numerous Greek and German letters which have no apparent relation to each other, entirely different symbols being used for homologous items in the prototype and model. Such a set of symbols makes it difficult to follow the literature which has been presented on this subject particularly because each writer uses his own set of letters. In keeping with the general scheme proposed by the author it might be well to use the symbol, l, for the basic scale ratio instead of α. The writer prefers the inverse of the ratio used by the author; that is, the ratio of the length, time period, velocity, etc., in the prototype to that in the model; this provides integers instead of fractions for the ratios and seems somewhat more convenient in application when dealing with a series of models of different size.

Frequently, it is not understood that although the usual mechanics are practically unlimited in applicability in so far as pure dynamic occurrences are concerned, the applicability of the mechanics of similitude has certain limitations. Not all motion occurrences can be imitated by means of a model, and therefore, not all dynamic problems met with in practice can be solved by means of the mechanics of similitude. In fact, relatively few motion occur-

NOTE.—The paper by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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[12a] Received by the Secretary December 9, 1933.

rences can be imitated accurately by means of a model. Obviously, therefore, the first step in the design of a model for the solution of a practical hydraulics problem necessitates a study of whether or not the problem can be treated at all by the method of similitude.

An analysis must be made to determine what physical forces cause the occurrence. The nature of these forces fixes the "model law" under which the model must operate, if it is to be similar to the occurrence in the prototype. The physical forces in the sense here used are those that cause acceleration of the particles forming the system concerned. Some of the more important are as follows: The earth's gravitational forces, frictional forces between liquid particles, capillary forces, elastic forces, normal and tangential forces (in solid bodies), and many others. Each one of these forces defines a model law; that is, the relation between the length ratio and time ratio for the prototype and model.

The most important physical forces that cause fluid motion are the earth's gravitational forces, friction between fluid particles, and capillary forces. It is seldom that any one of these is the sole cause of the occurrence. Consequently, perfect similitude is almost never obtainable; the motion of particles in the hydraulic model is almost never perfectly similar to that in the prototype. The solution of design problems by means of models, therefore, resolves itself largely into determining the limitations of the applicability of various models.

The degree of similarity obtainable between the model and the prototype depends upon the nature of individual circumstances. If the movements in the prototype and model are influenced by the action of only one type of physical force, the model occurrences may be made to take place in a mechanically similar fashion to the prototype occurrences. The model law that must be followed in order to obtain mechanical similitude may be determined from various equations. The most natural procedure seems to be to determine the relation of the basic ratios,[13] l, t, and f, in a manner somewhat as follows. The ratio of the inertia or mass accelerating forces may be represented by:

$$f = \frac{M_m\, B_m}{M_n\ B_n} \qquad (16)$$

in which, M_m and M_n represent the masses of homologous volumes in the model and prototype, respectively, and B_m and B_n represent the corresponding accelerations of these masses. (The writer prefers the inverse of the ratios herein presented; the ratios are set up to correspond to the author's paper for the sake of uniformity.) Inasmuch as, for geometric similarity, the masses vary as the cube of the linear dimension, the ratio, $\frac{M_m}{M_n}$, is equal to $\frac{\Gamma_m}{\Gamma_n} l^3$; also, since $B = \frac{d^2 X}{d T^2}$, the ratio, $\frac{B_m}{B_n} = \frac{l}{2}$. Therefore, in terms of

[13] See "Nomenclature" given by author in *Proceedings*, Am. Soc. C. E., November, 1933, p. 1417.

the scale ratios, the ratio of the mass accelerating forces may be represented by,

$$f = \frac{\Gamma_m}{\Gamma_n} l^3 \frac{l}{t^2} \frac{\Gamma_m}{\Gamma_n} \frac{l^4}{t^2} = \phi_1 (l, t) \qquad (17)$$

For similarity, the ratio of the inertia forces must be equal to the ratio of the physical forces producing the occurrence; hence,

$$f = \frac{F_m}{F_n} \qquad (18)$$

in which, F_m and F_n are the physical forces (such as the force of gravity) acting upon homologous masses in the model and prototype, respectively. This equation may be written in the form:

$$f = \frac{F_m}{F_n} = \phi_2 (l, t) \qquad (19)$$

The value of the force ratio, f, may be eliminated from Equations (17) and (19), thus obtaining the "model law" in question in the basic form,

$$t = \Omega (l) \qquad (20)$$

Here, an arbitrarily chosen length ratio, l (ratio of length between model and prototype), fixes the time ratio, t (ratio of time required for an occurrence to take place in the model to the time required for a corresponding occurrence to take place in the prototype). It will be observed that an infinite number of models (of different scales) are possible for this case in which only one physical force comes into consideration. It will also be noted, from Equation (17), that the model law is influenced by the ratio of the densities of the mediums used, respectively, for the model and prototype. Equation (20) is of general form for all model laws, provided only one physical force comes into consideration.

Consideration will not be given at this time to various model laws which might be derived from the foregoing equations; suffice it to use the force of gravity as the physical force causing the occurrence in order to exemplify the method of analysis. Equation (17) will be written in the form,

$$f = \frac{M_m B_m}{M_n B_n} = \frac{\Gamma_m}{\Gamma_n} \frac{l^4}{t^2} \qquad (21)$$

and Equation (19) in the form,

$$f = \frac{F_{gm}}{F_{gn}} = \frac{W_m v_m}{W_n v_n} = \frac{W_m}{W_n} l^3 \qquad (22)$$

in which, W_m and W_n are the unit weights of the mediums of the model and prototype. These unit weights may be represented by $W_m = \Gamma_m G_m$ and $W_n = \Gamma_n G_n$, in which, G_m and G_n are the homologous accelerations of gravity in the similar occurrences. The expressions, v_m and v_n, are homologous volumes. Of course, normally, G_m and G_n are equal, the value being 32.09 at the equator, 32.17 at the 45° parallel, and 32.25 at the poles.

By equating the two expressions for f given in Equations (21) and (22), and solving for the ratio of time periods:

$$t = \sqrt{l \frac{\Gamma_m \div W_m}{\Gamma_n \div W_n}} \qquad (23)$$

so that the model law (relation of time ratio to length ratio), if the force of gravity is the only physical force influencing the occurrence, is given by,

$$t = \sqrt{l \frac{G_n}{G_m}} \qquad (24)$$

This relation is referred to as the Froude model law. If G is the same for both prototype and model, Equation (24) takes the form,

$$t = \sqrt{l} \qquad (25)$$

A similar method of analysis might be used in determining the model law for other physical forces. Once having the model law one can readily determine the relation of homologous discharges, homologous velocities, homologous accelerations, etc., of the prototype and model.

However, as pointed out by Lieut. Vogel, ordinarily perfect similarity is not obtainable; this is due to the fact that more than one physical force influences the motion occurrences. It will be assumed, for example, that two physical forces effect the flow conditions, possibly the force of gravity and frictional forces between liquid particles. Three equations are then obtained between the force ratio, time ratio, and length ratio, as follows:

Ratio of inertia forces:

$$f = \phi_1 (l, t) \qquad (26)$$

Ratio of gravity forces:

$$f = \phi_2 (l, t) \qquad (27)$$

Ratio of frictional forces:

$$f = \phi_3 (l, t) \qquad (28)$$

Since there are three equations with three unknowns (the ratios, l, t, and f), none of the unknowns may be chosen arbitrarily as in the simple case of only one type of force influencing the occurrence; each unknown has a definite value. Therefore, when once the medium of the model is fixed, there is only one size model that will provide perfect mechanical similarity. If the medium in both prototype and model are the same (for example, if it is water in both cases) then the model must be of the same size as the prototype for perfect similarity.

Since perfect similarity in models is seldom if ever obtainable in the development of practical hydraulic designs, recourse must be had to certain approximations. This is particularly true in the case of working models of rivers and harbors. For these, in the apt words of the author, "the question is not whether distorted models will be used; it is rather how can the distortion that is certain to occur in every model be made least harmful?" What are the advantages and shortcomings of different types of distortion?

In river models, similarity based upon the empirical Manning formula apparently gives better agreement with experimental results than either Reynolds' model law or Froude's model law. In this case the predominant forces influencing the occurrence are fluid friction and the force of gravity. However, similarity cannot be completely satisfied, ordinarily, for either of these forces if the most workable model is to be obtained. Usually, in well-designed river models, Reynolds' number is smaller than in the prototype. In some laboratories, the practice has been to design such models to satisfy Froude's number as far as velocities are concerned and to limit the scale ratio in such a manner that turbulent flow will occur in the model; also, in the case of movable bed models, the scale ratio has been limited so that sediment transportation will occur. Table 2 indicates that Manning's formula gives a better value for the velocity and discharge than Froude's number.

The necessity of geometrical distortion becomes particularly evident when designing models of large alluvial rivers having relatively fine bed materials. In many cases it will be found necessary to make a model of a size covering several acres if recourse is not taken to some kind of geometrical distortion. The large size often imposed in order to obtain a geometrical non-distorted workable model would result in excessive cost of construction of the model and very cumbersome operation.

For broad river channels the hydraulic radius, for all practical purposes, is proportional to the mean depth of the channel; in the case of a model in which the vertical scale is different from the horizontal scale, therefore, the hydraulic radius will here be assumed equal to the mean depth. Using the author's nomenclature and Manning's formula as a basis, for a vertically distorted model the velocity ratio becomes:

$$v = \frac{d^{\frac{2}{3}} s^{\frac{1}{2}}}{n} \qquad (29)$$

If the amount of distortion is represented by $k = \dfrac{d}{l}$, it will be found that the velocity in the distorted model may be represented by:

$$v = \frac{1}{n} d^{\frac{2}{3}} \left(\frac{d}{l}\right)^{\frac{1}{2}} = \frac{1}{n} k^{\frac{7}{6}} l^{\frac{2}{3}} \qquad (30)$$

Thus, the velocity increases as the seven-sixth power of the distortion. This condition is particularly desirable inasmuch as the velocity in small scale models frequently becomes too low.

Vertical distortion also results in a definite increase in the tractive force of the stream in moving sediment along the bottom of the model river. Assuming the ratio of the tractive forces of the model and prototype to be represented by f, the ratio may be expressed by the linear ratios, by:

$$f = d\,s = d\,\frac{d}{l} = k^2\,l \qquad (31)$$

showing that the tractive force increases as the square of the vertical distortion. In a similar fashion, it may be shown that the ratio of the Reynolds' numbers for the model and prototype is represented by:

$$\frac{\mathsf{R}_m}{\mathsf{R}_n} = k^{\frac{13}{6}} l^{\frac{5}{3}} \qquad (32)$$

Reynolds' number thus increasing as the thirteen-sixth power of the distortion for a given horizontal scale ratio. This is a particularly desirable condition because the Reynolds' number of the model is ordinarily lower than that of the prototype in the case of river models. Distortion thus results in a degree of turbulence more nearly that of the prototype. These advantages of vertical distortion may be summarized as follows:

1.—The velocity in the model is increased by distortion, thus resulting in more suitable Reynolds' numbers and making possible more precise measurements in consequence of the increased discharge.

2.—Tractive forces (and consequent suitability of models for use in experiments involving movable beds) are increased markedly.

3.—Great economies arise in the cost of construction and operation of the model because the necessary horizontal scale may thereby be greatly reduced.

Counterbalancing these advantages are a number of disadvantages, particularly where the amount of distortion attempted is very large. For various reasons accurate similarity of the bed elevations in the distorted model as compared to the prototype are not possible. This is particularly true where the erosion in the prototype causes the bottom to stand at approximately the natural under-water angle of repose of the sedimentary material. For example, in Fig. 4 let the shaded area represent the idealized scour

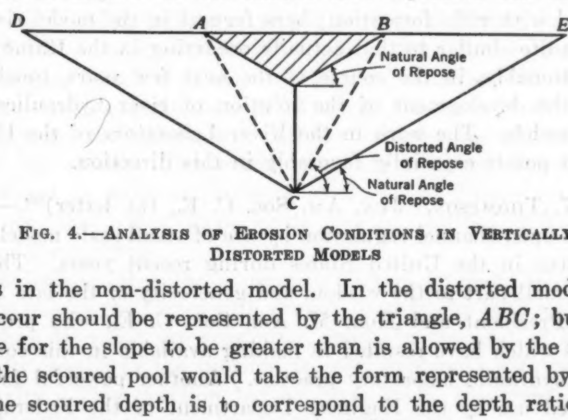

FIG. 4.—ANALYSIS OF EROSION CONDITIONS IN VERTICALLY DISTORTED MODELS

that occurs in the non-distorted model. In the distorted model the corresponding scour should be represented by the triangle, *ABC*; but, since it is not possible for the slope to be greater than is allowed by the natural angle of repose, the scoured pool would take the form represented by the triangle, *DEC*, if the scoured depth is to correspond to the depth ratio. It is conceivable, therefore, that where the vertical distortion is great, considerable deviation between the results of an undistorted model and the conditions in the prototype might obtain. A similar analysis could be made in the case of sedimentation; in fact, it might be shown that the model could indicate erosion where sedimentation would occur in the prototype.

Other conditions brought about as a result of distortion which might be harmful are the formation of fixed eddies (frequently referred to as "rollers"). Distortion may result in the elimination of these fixed eddies in some localities and the introduction of some in other localities, inasmuch as there are limits to the amount that a "roller" may be distorted without breaking it into counteracting "rollers."

The foregoing are some of the shortcomings that cannot be eliminated, nor can their effect be determined accurately without comparative experiments in the individual cases concerned. The observer must be constantly alert to avoid making erroneous deductions as a result of his experiments on models constructed to vertically distorted scales.

It has sometimes been suggested that the proper type of distortion consists in merely increasing the slope of the model—that is, by tilting the model to a slope steeper than that of the prototype while the horizontal length ratio and vertical length ratio remain equal. Such an arrangement has certain advantages in that it brings about the desired greater velocities and greater transporting force (although not to as marked a degree as vertical distortion) and greatly eliminates the disadvantage of producing "rollers" where they should not occur. On the other hand, in the case of a broad river model, inasmuch as the flow takes place in a serpentine rather than in a rectilinear direction, the model would have to be warped in order to obtain uniform distortion in slope.

In connection with experiments concerning the regimen of the Upper Rhine River, Dr. Theodore Rehbock, of Karlsruhe, Germany, found that certain advantages were obtained by using a sedimentary material composed of volcanic ash. The specific gravity of this material was considerably less than ordinary sand, being about 1.6. In this material no difficulty was encountered with riffle formation; bars formed in the model river channel in a manner quite similar to that actually occurring in the Rhine River.

Unquestionably, in the course of the next few years, much is to be expected in the development of the solution of river hydraulics problems by means of models. The work in the River Laboratory of the U. S. Engineer Department points especially favorably in this direction.

PAUL W. THOMPSON,[14] JUN. AM. SOC. C. E., (by letter)[14a].—The study of problems in open-channel regulation by use of small-scale models has received great impetus in the United States during recent years. This impetus is due in no small part to the wisdom and generosity of the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., who provided for the Fellowships which have resulted in making available in this country the best of foreign hydraulic laboratory practice. Another powerful factor has been the increased use by the Engineer Department of the Federal Government of the small-scale model as an aid in the solution of certain problems arising in the administration of the rivers and harbors of the country. The U. S.

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Waterways Experiment Station at Vicksburg, Miss., where most of the model studies for the Engineer Department are made, has been, for the past few years, among the most active laboratories in the world. It is encouraging to note that many of the principles followed and developed at that Station have been made available through the medium of this paper, which is a valuable addition to the literature of experimental hydraulics.

The author's remarks on the effects and importance of turbulence in open-channel models will serve to clarify this little understood subject. Turbulence is that fortunate condition of flow which justifies the assumptions that: (*a*) The forces of internal friction in the fluid are negligible; and (*b*) the force of gravity is predominant. The consistency of these two assumptions with the requirements of the law of Reeche-Froude (generally called the law of Froude) will be observed at once. The observation is substantiated by consideration of the facts that: (1) In turbulent flow, velocity is a function of the square root of the depth; while, similarly, (2) under the law of Froude, the velocity scale is a function of the square root of the depth scale.

The evaluation of a turbulence index as given by Equation (15) is a noteworthy addition to the list of practical aids to the experimentalist. The value of $Z = 0.02$, should be considered as an absolute permissible minimum. In most of the open-channel models at the U. S. Waterways Experiment Station the product of the actual mean velocity by the actual mean depth has proved to be several times this value. Turbulence is a comparative phenomenon—that is, there are degrees of turbulence. Thus, in a regimen in which the value of Z is, say, 0.06, the effects of the forces of internal friction will be more nearly negligible than in a regimen in which Z has a value less than 0.6. Short of going into "shooting flow," the experimentalist will seek as high a degree of turbulence as possible, since he will try to render the effects of internal friction as nearly negligible as possible.

Lieut. Vogel has touched on most of the practical difficulties that confront the small-scale model experimentalist. Perhaps the most important of these difficulties concerns the adjustment, and the need for the adjustment, of the roughness of the model. The writer, who has been privileged to co-labor with Lieut. Vogel on much of the work at the U. S. Waterways Experiment Station, believes that a few more words may be justified in regard to this matter of model roughness.

The basic decision that the experimentalist must make in planning his model study concerns the forces with respect to which he is to seek similitude. The author has discussed the reasoning which, in the case of open-channel models, usually results in the decision to seek similitude with respect to the force of gravity. This decision having been made, the question of model roughness immediately assumes important proportions.

The decision to seek similitude with respect to the force of gravity presumes, in effect, that water is a perfect fluid. The result is the law of Reeche-Froude; namely,

$$t = \sqrt{d} \qquad (33)$$

From this law follows immediately the relations expressed by Equations (2) and (3) of the paper. Consider, now, the difficulties that arise in an attempt to realize the fundamental relationships expressed by Equations (2), (3), and (33).

Assume that flow in an open channel in Nature is in accordance with the Chezy law:

$$V_n = C_n \sqrt{D_n\, S_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

and that flow in a model of that channel also accords with that law, as follows:

$$V_m = C_m \sqrt{D_m\, S_m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

It immediately follows, using the nomenclature of the author, that,

$$v = c \sqrt{ds} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Thus, if the fundamental relationship of Equation (2) (Froude's law) is to obtain, it is necessary that,

$$c \sqrt{s} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The experimentalist will endeavor to maintain an "undistorted' slope; that is, he will endeavor to keep s equal to unity. This is true because of the impossibility of analyzing completely all the effects due to any form of distortion. However, if s is to equal unity, then according to Equation (37), c must equal unity. Since c is simply the quotient of the C for the model (C_m) divided by the C for Nature (C_n), this is tantamount to stating that,

$$C_m = C_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Consider now the position of the experimentalist who is designing an open-channel model, and who is aiming for similitude with respect to the force of gravity. He knows that the accuracy of his results will be a function of the degree to which he can satisfy the law of Equation (38). The degree of his success in satisfying this law must rest on: (*a*) The accuracy to which the value of C_n may be determined; and (*b*) the degree of control, or manipulation, that may be exercised on C_m. The experimentalist will proceed to study all the data available concerning the laws of flow in the prototype, and he will thus arrive at his "best value" for C_n. He will then consult his own records, and the records of others, in an endeavor to determine just how he may insure a value of C_m which approaches the "best value" for C_n. Thus, as an idealized illustration, suppose that the experimentalist has determined that $C_n = 40$. From his own files and from the files of others, he finds records of models, similar to the one he contemplates, as follows: (*a*) For Model *A*, made of rough mortar, $C_m = 30$; (*b*) for Model *B*, made of glazed plaster of Paris, $C_m = 46$; and (*c*) for Model *C*, made of smooth mortar, $C_m = 41$. Obviously, the experimentalist decides to construct his model of smooth cement mortar, since, in the light of past experiences, it offers the best chance of satisfying the law of Equation (38).

Actually, the problems will never be so simple of solution as indicated in the illustration. First, the "best value" for C_n often is not a very good value. Indeed, in many instances—especially those in which proposed construction will exert great influence on the value of C_n—the "best value," in reality, may be the "best estimate." Second, such records of model roughness coefficients as are now available are scarcely ever extensive enough to justify absolute dependence on them; in fact, in many instances, such records as are available form little more than general indications or guides. In such instances, the experimentalist must keep before him the factors that influence the value of C. Thus, taking the Manning relationship (because it is used by the author), the experimentalist considers that $C \propto \frac{D^{\frac{1}{6}}}{N}$. Herein are indicated the effects of the reduced linear dimensions of the model. Thus, N is a value that increases with increasing roughness; but D is a value that decreases as the linear scale ratio, model to Nature, decreases. It can thus be seen that, if Equation (38) is to be satisfied, the model, as it is made progressively smaller, must be made correspondingly smoother.

After his model is built, the experimentalist will proceed to "adjust" it; that is, he will run many tests, under various conditions, and obtain, thereby, actual values for C_m. He will be able, by smoothing or roughening the model, to manipulate his values within reasonable limits. He will continue his "adjustments" until he is satisfied that the requirements of Equation (38) are as well fulfilled as is practicable.

This discussion has concerned itself with the problem of satisfying Froude's law in the case of an "undistorted" model; but, as pointed out and explained by the author, most of the problems arising from the great alluvial rivers of this country require models in which the horizontal linear scale ratio is considerably smaller than the corresponding ratio for vertical dimensions. In building such models, it is customary (although not essential) to make the slope of the model greater than the slope of the prototype by the value, $\frac{d}{l}$. In other words, usually, $s = \frac{d}{l}$. The significance of such a condition, as effecting the subject under discussion, is apparent from a consideration of Equation (37). Thus, consider the values given for Model F, in Table 2: $l = \frac{1}{720}$; and, $d = \frac{1}{72}$. For Model F, $s = 10$; hence, in this particular case, if the law of Froude were to be satisfied, $c = \frac{1}{\sqrt{10}}$ or,

$$C_m = \frac{1}{\sqrt{10}} C_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Considering that the value of C decreases as roughness increases, Equation (39) serves to explain an apparent anomaly. Thus, the fact that the

dimensions of individual roughnesses cannot be reduced to the same extent as other dimensions, would justify the expectation that small-scale models, in general, would be too rough. However, in the case of distortion of scales, the slope factor enters, as illustrated in Equation (39), and often operates to make the model too smooth. A case in point is Model *F*, already noted. Table 2 shows that the actual discharge scale of the model was $\frac{1}{280\,000}$, while, according to the Froude law, it should have been $\frac{1}{440\,000}$. Obviously, the model was too smooth; that is, C_m was too large.

The author presents, as Equation (10), an expression for discharge scale obtained by the simple process of applying the Manning formula to the discharges of model and Nature, respectively, and dividing the former result by the latter. Implicit in his operations are the assumptions of $n = 1$, and $s = \frac{d}{e}$. Obviously, if the Froudian law is to hold, peculiar and definite relationship must exist between the d and the l of Equation (10). This relationship may be easily determined by writing the Froudian expression for discharge scale, Equation (3), and solving it simultaneously with Equation (10). This operation was reported in another connection,[15] wherein it was found that, if $d = l^{\frac{7}{4}}$, there is no conflict between Equation (10) and the Froude law. This requirement has been called the "law of compatibility." Making the same assumptions that the author makes in deriving Equation (10), ($n = 1, s = \frac{d}{e}$), and inserting the Manning expression for C in Equation (37), it follows as before that $C = l^{\frac{1}{4}}$. Thus, the law of compatibility is an adaptation, under certain given conditions, of the general requirement, c , $s = 1$.

In this connection it can be noted that a judicious manipulation of the slope ratio, s, might serve as a means of obtaining satisfaction of Equation (38). This notation is entirely correct, but it should be observed that such manipulation introduces the indeterminate effects of a distortion in the model results. Furthermore, it should be observed that the slope ratio must be determined before the model is built, and may not be altered after construction. Since it is impossible to foresee exactly the effects of reduced dimensions, altered slopes, etc., it follows that manipulation of the slope ratio alone will not insure satisfaction of the law. Resort must still be had to the methods already described.

It must not be concluded that failure to obtain exact satisfaction of the law of Equation (38) invalidates the model results. Such failure, of course, does affect the results, and must be taken into careful consideration when data from the model study are analyzed and interpreted. A small-scale model represents at best a series of compromises—a structure in which theoretical considerations have conflicted, at many points, with practical possibilities. The word, model, is itself almost too exact an appellation to

[15] *Civil Engineering;* August, 1932, p. 467.

apply to the small-scale replica of a natural stream. More strictly appropriate would be some title, such as "approximate reproduction."

The writer is a firm believer in the advisability of the use of small-scale models in the solution of open-channel problems where large sums of money are involved. His belief, indeed, is so firm that he does not deem it necessary to overstate the case in favor of the use of models by a single iota. The open-channel laboratory can stand on its own feet, and is more and more able, as time goes on, to point to the record books for its justification. With all its limitations, the small-scale model is still a valuable tool for the river hydraulician who understands its use.

Lieut. Vogel has been in the very front rank of those who have advanced the technique of hydraulic laboratory practice in this country to the high plane it now occupies. His paper will have the undivided praise and interest of all those who are interested in the great science of experimental hydraulics.

Ralph W. Powell,[16] M. Am. Soc. C. E. (by letter)[16a].—The great increase during the last few years in the use of models for the solution of various problems connected with rivers, makes this paper very timely, and the author deserves the thanks of the profession for making the results of his experience available.

There are only a few minor criticisms. Where so many quantities are involved there seems a distinct advantage in making use of the Greek alphabet, especially where such use is already familiar. It would seem better not to break with such established usage as ρ for density, μ for coefficient of viscosity, and ν for coefficient of kinematic viscosity. It must be discouraging to those who are working for uniformity in the use of symbols to see one of the best established ones —g for the acceleration of gravity— replaced by G. Furthermore, M is called "the viscosity modulus for open channels." It would seem that "critical value of Reynolds' number for open channels" would better express the idea. However, the writer has endeavored to abide by the author's nomenclature in what follows.

The statement preceding Equation (10), that small-scale models of cement mortar and intermediate-scale models of sand, have about the same coefficient of roughness as the natural river, is interesting. Certainly Kutter and Ganguillet, Bazin, and Manning meant the coefficient of roughness in their formulas to depend only on the material, and to be independent of the size of the channel cross-section; but it is also probably true that none of the formulas actually holds over a wide range of sizes. The increasing use of small-scale models gives the question an importance it had not had before, and creates an urgent demand for a new and better formula for channel flow. As a preliminary step in that direction, the writer offers the following theoretical considerations.

Suppose that the average velocity over the cross-section of the channel is V, and that it depends on the following seven factors only: S, the surface

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[16a] Received by the Secretary January 8, 1934.

slope; *D*, the mean depth = cross-sectional area divided by surface width; *W*, the surface width; *G*, the acceleration of gravity; *p*, the density of the liquid; *u*, the coefficient of viscosity of the liquid; and, *E*, a measure of the roughness, such as the average diameter of the sand grains, the average height of the projections which produce the roughness of a cement model, or some such quantity. Then, the formula desired will be of the form:

$$f\,(V, S, D, W, G, p, u, E) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

By a method developed independently by D. Riabouchinsky[17] and by Edgar Buckingham,[18] this can be replaced by the equation:

$$\phi\,(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which all of the π's are dimensionless, and, $\pi_1 = V^{x_1}\,D^{y_1}\,u^{z_1}\,G^{-1}$; $\pi_2 = V^{x_2}\,D^{y_2}\,u^{z_2}\,p$; $\pi_3 = \dfrac{E}{D}$; $\pi_4 = \dfrac{W}{D}$; and, $\pi_5 = S$.

The dimensions of the quantities in Equation (40) are:

$$\begin{array}{ll} [V] = [L\,T^{-1}] & [G] = [L\,T^{-2}] \\ [S] = [L^0] \text{ (dimensionless)} & [p] = [M\,L^{-3}] \\ [D] = [L] & [u] = [M\,L^{-1}\,T^{-1}] \\ [W] = [L] & [E] = [L] \end{array}$$

The values of the x's, y's, and z's are found by applying the principle of dimensional homogeneity to the expressions for the π's; π_1 gives,

$$[\pi_1] = 0 = [L\,T^{-1}]^{x_1}\,[L]^{y_1}\,[M\,L^{-1}\,T^{-1}]^{z_1}\,[L\,T^{-2}]^{-1} \ldots\ldots\ldots (42)$$

Equating the exponents of M:

$$z_1 = 0$$

Equating the exponents of T:

$$-x_1 - z_1 + 2 = 0$$

therefore, $x_1 = 2$.

Equating the exponents of L:

$$x_1 + y_1 - z_1 - 1 = 0$$

therefore, $y_1 = -1$, and $\pi_1 = \dfrac{V^2}{D\,G}$. This is "Froude's number" and is usually represented by the symbol, **F**.

The expression for π_2 gives:

$$[\pi_2] = 0\,[L\,T^{-1}]^{x_2}\,[L]^{y_2}\,[M\,L^{-1}\,T^{-1}]^{z_2}\,[M\,L^{-3}] \ldots\ldots\ldots (43)$$

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17 "Methode des Variables de Dimensions Zero," par D. Riabouchinsky, *L'Aerophile*, September 1, 1911.

18 "On Physically Similar Systems," by Edgar Buckingham, *Physical Review*, Vol. 4, pp. 345–376 (1914); also, "Model Experiments and the Forms of Empirical Equations," *Transactions*, Am. Soc. Mech. Engrs., Vol. 37, pp. 263–296 (June, 1915); and "Notes on the Method of Dimensions," *Philosophical Magazine*, 6th Ser., Vol. 42, (November, 1921), pp. 696–719.

Equating the exponents of M,

$$z_2 + 1 = 0$$

therefore, $z_2 = -1$.

Equating the exponents of T,

$$-x_2 - z_2 = 0$$

therefore, $x_2 = 1$.

Equating the exponents of L,

$$x_2 + y_2 - z_2 - 3 = 0$$

therefore, $y_2 = 1$ and $\pi_2 = \frac{V\,D\,p}{u}$. This is Reynolds' number, R. If, similarly, $\pi_3 = \frac{E}{D}$ is represented by E, and $\pi_4 = \frac{W}{D}$ by W, Equation (41) becomes ϕ ($\mathsf{F}, \mathsf{R}, \mathsf{E}, \mathsf{W}, S$) = 0, or,

$$S = \Phi\ (\mathsf{F}, \mathsf{R}, \mathsf{E}, \mathsf{W}) \qquad (44)$$

By experiment it is known that F is by far the most important of the four factors, and that as far as it is concerned the function is of the simplest sort; therefore,

$$S = k\,\mathsf{F} = \frac{k\,V^2}{G\,D} \qquad (45)$$

in which $k = \theta$ ($\mathsf{R}, \mathsf{E}, \mathsf{W}$). The classical formulas are in terms of R, the hydraulic radius, instead of D, the mean depth, but in this discussion the writer follows the author in using them more or less synonymously. Then, the Chezy formula is Equation (45), with $\frac{G}{k} = C^2$. Bazin's formula,

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{R}}}$$

, would reduce to:

$$k = \frac{G\left(0.305 + \frac{1.104\,m}{\sqrt{R}} + \frac{m^2}{R}\right)}{7\,569} \qquad (46)$$

which is obviously not dimensionally homogeneous and, therefore, cannot be the ideal formula that is being sought. The same applies to Kutter's formula. Manning's formula may be put in the form, $S = \frac{N^2\,V^2}{2.208\,R^{\frac{4}{3}}}$, which is Equation (45), if $R = D$ and $k = \frac{N^2\,G}{2.208\,D^{\frac{1}{3}}}$. This shows that the dimensions of Manning's N are $[T\ L^{-\frac{1}{6}}]$ and that his formula cannot be expected to give

the same value of N for the same surface in different sizes. The Williams and Hazen formula, $V = 1.318\ c\ R^{0.63}\ S^{0.54}$, reduces to Equation (45) if $R = D$ and $k = \dfrac{G}{1.737\ c^2\ D^{0.26}\ S^{0.08}}$. This makes the dimensions of the Williams and Hazen $c = [T^{-1}\ L^{0.37}]$, so that it would be expected to vary with the size slightly more than Manning's N.

Until the true form of the Φ-function in Equation (44) is known the simplest assumption to make is that it is of the form, $S = k\ \mathsf{F}\ \mathsf{R}^{-x}\ \mathsf{E}^{y}\ \mathsf{W}^{z}$, which reduces to:

$$S = \frac{k\ K^x\ E^y\ W^z\ V^{2-x}}{G\ D^{1+x+y+z}} \qquad (46)$$

in which, K is the coefficient of kinematic viscosity $= \dfrac{u}{p}$ and k is an absolute dimensionless constant. From the analogy with pipes it is quite certain that x is between 0.00 and 0.10; but a great amount of careful research would be required to get a more accurate value and to find y and z. Furthermore, it might be found that no equation of the form of Equation (46) would accurately represent the facts, because x is probably not a constant for all roughnesses, and probably the velocity distribution, as determined by bends in the channel, etc., is quite important; and it has not been brought into the formula at all.

If, however, it is assumed that at least Equation (46) will approximately represent the facts, the requirement for similarity between model and prototype would be:

$$\frac{S_m}{S_n} = \frac{d}{l} = \frac{K^x{}_m\ E^y{}_m\ W^z{}_m\ V_m^{2-x}\ D_n{}^{1+x+y+z}}{K^x{}_n\ E^y{}_n\ W^z{}_n\ V_n{}^{2-x}\ D_m{}^{1+x+y+z}} \qquad (47)$$

If $K_m = K_n$, $\dfrac{d}{l} = \dfrac{e^y\ l^z\ v^{2-x}}{d^{1+x+y+z}}$, or,

$$v^{2-x} = \frac{d^{2+x+y+z}}{l^{1+z}\ e^y} \qquad (48)$$

For a non-distorted model, in which $l = d$,

$$v^{2-x} = \frac{d^{1+x+y}}{e^y} \qquad (49)$$

Multiplying by the area scale, $a = l\ d$, to get discharge, q for the distorted model is expressed by,

$$q^{2-x} = \frac{d^{4+y+z}\ l^{1-x-z}}{e^y} \qquad (50)$$

and for no distortion,

$$q^{2-x} = \frac{d^{5-x+y}}{e^y} \qquad (51)$$

Assuming that $x = 0.04$, Equation (50) becomes,

$$q^{1.96} = \frac{d^{4+y+z}\ l^{0.96-z}}{e^{y}}$$

or,

$$q = \frac{d^{0.51(4+y+z)}\ l^{0.51(0.96-z)}}{e^{0.51\,y}} \qquad (52)$$

Then n in the author's Equation (8) will be:

$$n = l^{0.010+0.51z}\ d^{0.127-0.51\,(y+z)}\ e^{0.51y} \qquad (53)$$

The values of n as computed from Table 2 are:

Test	Value of n	Test	Value of n
A	0.875	D	0.987
B	1.075	E	1.056
C	1.002	F	0.986

Using the values of n, l, and d for Tests A, B, C, and D, and solving by the method of least squares for the most probable values of y, z, and e, gives $y = 0.40$, $z = -0.85$, and $e = 1:2\,800$; and $n = l^{-0.424}\ d^{0.357}\ e^{0.204}$. Then, Tests E and F give $e = 1:374$ and $1:523$, respectively. Taking the average as $1:450$, it can be said that these tests indicate that the sand was about 6.25 times as coarse as the equivalent size of the cement surface, and that the natural river was 450 times as coarse as the sand. Undoubtedly, these values are incorrect, but they are near enough within the range of probability to indicate that this method of attack, supported by extensive tests on channels of different scales and distortions, and definite roughnesses, might lead to a satisfactory formula for the flow in open channels, and, therefore, to a true law of model similarity.

K. D. NICHOLS,[19] ESQ. (by letter)[19a].—Although this is a valuable contribution to the science of experimental hydraulics and covers the entire field relating to the solution of river hydraulic problems in the laboratory in an excellent manner, there are many points that should be explained more completely.

A pertinent case relates to the selection of a suitable sand for use in a movable bed model. Under the heading "Experiment Involving Movable Beds," Lieut. Vogel has epitomized in six steps the procedure to be followed in order to insure proper movement of the model bed material. This procedure insures movement of sand in the model. However, it gives no index of the amount of movement to be expected. An improved method is to select a sand that, in addition to fulfilling all the requirements as listed by Lieut. Vogel, will also give adequate movement. This selection must be based on flume tests to determine the rate of movement for any given tractive force.

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[19] 2d Lieut., Corps of Engrs., U. S. Army; Asst. Director, U. S. Waterways Experiment Station, Vicksburg, Miss.

[19a] Received by the Secretary January 15, 1934.

Fig. 5 and Table 4 illustrate the mechanical sieve analyses of four sands that have been tested in a tilting flume at the U. S. Waterways Experiment Station. The theoretical critical tractive force has also been determined by

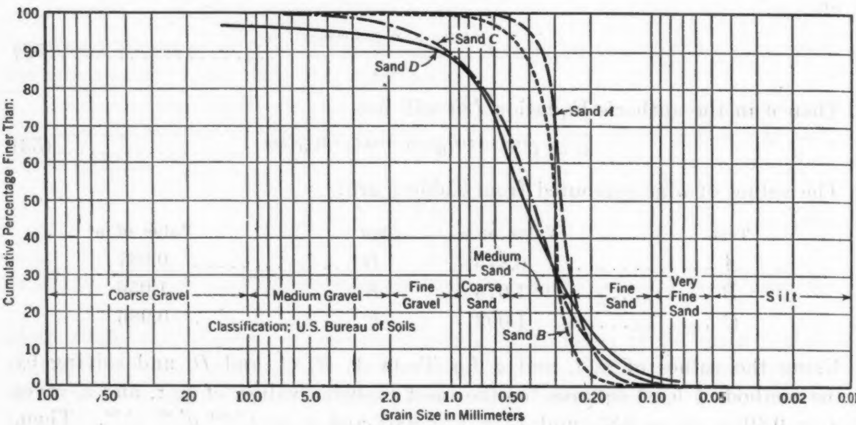

FIG. 5.—MECHANICAL ANALYSIS DIAGRAM

formula for each sand and is listed. On the basis of the theoretical critical tractive force necessary to insure movement Sand *A*, with the least tractive force required for general movement, is the best sand, followed in order by Sands *B*, *C*, and *D*.

TABLE 4.—CHARACTERISTICS OF SAND SPECIMENS

Item No.	Symbol	Definition	Sand Specimen: *A*	*B*	*C*	*D*
1	E	Mean grain diameter..........................	0.2842	0.3470	0.5584	1.0257
2	D_m	Median grain diameter..........................	0.2714	0.3367	0.3984	0.4358
3	B	Sand uniformity factor..........................	0.6482	0.6428	0.3222	0.1460
4	F	Critical tractive force, in grams per square meter..........................	11.91	14.84	46.80	193.19
5	j_1	Specific gravity of bed material..........................	2.63	2.65	2.62	2.65

Fig. 6 illustrates the results of tests made in a tilting flume to determine the rates of movement for the four sands. The tests indicate that Sand *C* has a much higher rate of movement for any given tractive force than either Sand *A* or Sand *B*. Sand *D*, while having in general a more rapid rate of movement than Sand *C*, requires a higher tractive force for the beginning of general movement.

One of the chief advantages of using a more rapidly moving sand is the saving in time required for the operation of a river model. Consider, for example, a model of a reach in the Mississippi River with scales, as follows: $l = \dfrac{1}{1\,000}$, $d = \dfrac{1}{125}$, and $s = 17.4$. The use of Sand *C* in lieu of Sand *A*

reduces considerably the time of operation of the model to represent a given time in the prototype. Table 5 compares the rates of movement of Sands *A* and *C* to be expected at different stages. The values in Column (2) are the

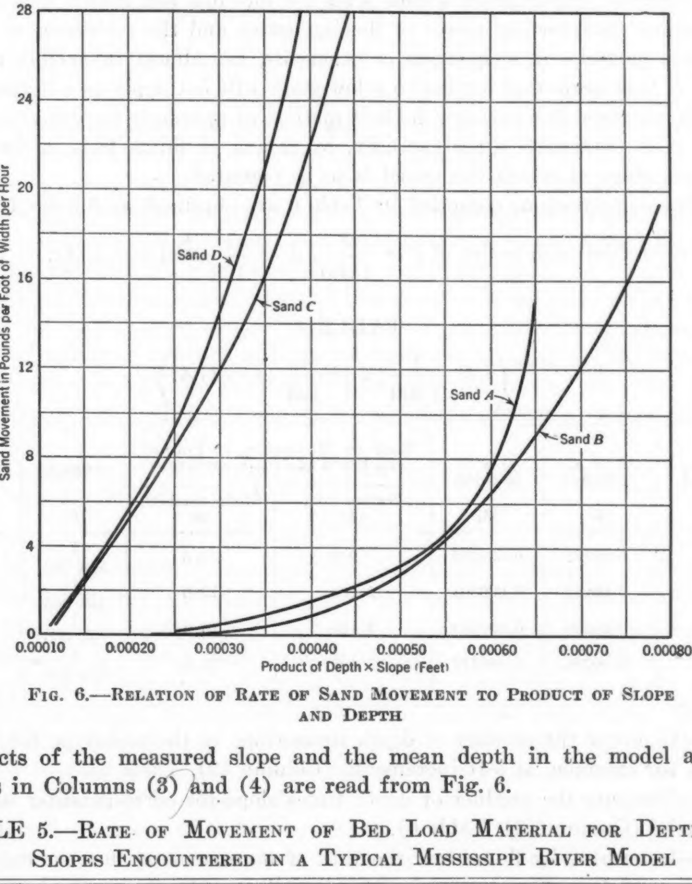

FIG. 6.—RELATION OF RATE OF SAND MOVEMENT TO PRODUCT OF SLOPE AND DEPTH

products of the measured slope and the mean depth in the model and the values in Columns (3) and (4) are read from Fig. 6.

TABLE 5.—RATE OF MOVEMENT OF BED LOAD MATERIAL FOR DEPTHS AND SLOPES ENCOUNTERED IN A TYPICAL MISSISSIPPI RIVER MODEL

STAGE	$D \times S$	RATE OF MOVEMENT, IN POUNDS PER HOUR PER FOOT OF WIDTH		Ratio of movement, $C \div A$
		Sand *A*	Sand *C*	
(1)	(2)	(3)	(4)	(5)
0	0.000252	0.2	8.5	42.5 : 1
5	0.000308	0.6	12.0	20.0 : 1
10	0.000360	1.1	16.9	15.4 : 1
15	0.000412	1.8	23.5	13.1 : 1
20	0.000458	2.5	28.5	11.4 : 1
25	0.000504	3.3	....	......
35	0.000596	7.2	....	......

It should be noted that the ratio of the rate of movement of Sand *C* to Sand *A* is not a constant for different stages. For a 5-ft stage twenty times as much sand moves if Sand *C* rather than Sand *A* is used, whereas for a 15-ft

stage the ratio of the rates of movement is only 13.1 : 1. Similarly, for other sands, the ratio of rates of movement varies according to some as yet unknown law. This variance for any two sands emphasizes Lieut. Vogel's statement that formulas representing a time scale for movable bed models based on the theoretical transporting power of flowing water and the resistance to motion of grain particles in suspension or otherwise, are almost invariably misleading. A time scale that applies to a low stage will not apply to a higher stage.

This suggests that perhaps the best method of approach for deriving a time scale is to determine experimentally, by means of flume tests, a time scale for each stage at which the model is to be operated.

The computations compiled in Table 6 are obtained, as follows (data are for river model with scales of $l = \frac{1}{1\,000}$, $d = \frac{1}{125}$, and $s = 17.4$):

TABLE 6

$$\left(l = \frac{1}{1\,000}; d = \frac{1}{125}; s = 17.4.\right)$$

Stage, in feet	$D \times S$, in Nature	$D \times S$, in model	Rate of Movement, in Pounds per Hour per Foot, of Width		q (sand)	t (sand)
			Nature sand	Model sand (C)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.....	0.00181	0.000252	900	8.5	$\frac{1}{106\,500}$	$\frac{1}{1\,170}$
5.....	0.00222	0.000308	2 200	12.0	$\frac{1}{182\,000}$	$\frac{1}{690}$
10.....	0.00259	0.000360	4 150	16.9	$\frac{1}{244\,000}$	$\frac{1}{510}$
15.....	0.00296	0.000412	7 500	23.5	$\frac{1}{323\,000}$	$\frac{1}{390}$

1.—Compute the product of depth times slope in the prototype for various stages, for example, at 5-ft increments (Column (2), Table 6).

2.—Compute the product of depth times slope for corresponding stages in the model (Column (3), Table 6).

3.—Determine by flume test the rate of movement of the prototype sand for values of $D \times S$ computed in Step 1 (Column (4), Table 6).

4.—Determine by flume test the rate of movement of the model sand for values of $D \times S$ computed in Step 2 (Column (5), Table 6).

5.—Determine the discharge scale value for bed-load movement (Column (6), Table 6), by the formula,

$$q \text{ (sand)} = \frac{\text{Pounds of model sand per foot of width per hour}}{\text{Pounds of Nature sand per foot of width per hour}} \times l$$

6.—Compute the time scale for each stage, by the formula:

$$\text{Time scale, } t \text{ (sand)} = \frac{\text{Volume scale}}{\text{Discharge scale}}$$

For example (see Column (7), Table 6), for the 5-ft stage,

$$t \text{ (sand)} = \frac{l^2 d}{q \text{ (sand)}} = \frac{\dfrac{1}{125\,000\,000}}{\dfrac{1}{182\,000}} = \frac{1}{690}$$

Table 6 illustrates the variation in time scale for various stages, for a typical movable bed model. It should be noted that the values in Column (4), Table 6, have been obtained by extrapolation of existing data for a Mississippi sand and probably are only approximately correct. However, it is possible to determine the proper data experimentally and more work should be done along this line as soon as possible. These values should be obtained by flume tests for the particular sand in the prototype.

Having obtained, experimentally, the time scale for the various stages at which the model is to be operated, a model operating chart representing a cycle of one year is computed on the basis of allowing a time interval for each stage equal to the product obtained by multiplying the time scale by the time duration of that stage in the prototype during a typical or average year.

The procedure described herein is only one of various methods of attacking this difficult problem.

FRANK W. EDWARDS,[20] JUN. AM. SOC. C. E. (by letter)[20a].—As the possibilities offered by model studies have gradually become apparent to engineers in the United States, the application of the principle of hydraulic similitude has attracted more attention. A number of papers have been written on the subject, some of which have been highly theoretical. These have their place in engineering literature, of course, but it has seemed that the purely theoretical papers should be supplemented with practical suggestions for actual model design, operation, and interpretation. The author has furnished the needed supplement in his practical paper, the purpose of which, as set forth in its "Synopsis," has been fulfilled.

It is the purpose of this discussion to bring out a few points that may be of interest to engineers engaged in model study. The following points are discussed: First, the author's Equation (10); second, the possibility of selecting the horizontal and vertical scales, for a proposed model, in such a manner that the discharge ratio, q, will more nearly satisfy both the gravity and friction formulas; and third, the feasibility of controlling, within limits, the roughness factor in the model.

In Table 2 of the author's paper, Equation (10) has been compared with the actual discharge ratio, as determined experimentally in several models. The relatively close agreement between the ratio, as determined by the formula and by experimentation, is rather surprising. This formula is restricted in three ways: (*a*) The roughness factors in the model and its prototype must be equal; (*b*) the slope of the model must not be distorted

[20] Vicksburg, Miss.

[20a] Received by the Secretary January 16, 1934.

artificially (that is, the slope in the model should equal the slope in Nature multiplied by the scale distortion); and (*c*) the cross-sections of both the model and its prototype must be such that the hydraulic radius can be replaced by the mean depth without causing appreciable error. It is possible to meet the requirements imposed by the first two restrictions, but the third offers difficulties. In the majority of cases the substitution of d, the scale ratio for the depth, for r, the scale ratio for the hydraulic radius, would introduce a considerable error. In a cross-section of a river which is wide in comparison with its depth, the hydraulic radius may be replaced by the mean depth. The resulting error will be negligible. When this same section is reproduced in a model having a distorted scale, the depth, which is greatly exaggerated, is no longer insignificant when compared with the width. In some cases, when a large distortion is used in a model, the mean depth in the model cross-section might equal twice the value of the hydraulic radius for the same section. This reasoning suggests that, in the majority of cases, the actual ratio, r, as determined from cross-sections in the model and in Nature, should be used in the formula for interpreting certain results of model operation. The formula would then be written,

$$q = l^{\frac{1}{2}}\, d^{\frac{3}{2}}\, r^{\frac{2}{3}} \qquad (54)$$

It should be noted, also, that using the ratio, r, introduces more difficulties. This ratio will not be the same for all stages of the river. For the lower river stages, R_m, the hydraulic radius in the model, would approach D_m, the mean depth in the model; but, more often than not, the high stages are the subject of experimental investigation. For certain types of problems, the scale ratios should be chosen to keep the variation of the ratio, r, within definite limits for all the river stages to be simulated in the model.

As to the second point: When a model study involves, at the same time, flow through an open channel and flow over a structure, such as a spillway or submerged wing-dam, both gravity and friction should be considered. The author's Equation (3) follows from Froude's law, and Equation (10) from the Manning formula. In a specific problem, the correct ratio for r could be used in the formula instead of replacing r with d. To satisfy both laws, $l\, d^{\frac{3}{2}} = l^{\frac{1}{2}}\, d^{\frac{13}{6}}$, and,

$$d = l^{\frac{3}{4}} \qquad (55)$$

This means, according to the Manning formula, that the depth scale must equal the length scale raised to the three-fourths power, if both the gravity and friction formulas are to be satisfied.

Using a similar procedure with the Kutter formula as a basis, it is learned that the value of the exponent for l varies for each specific problem. For ordinary models, having scale ratios within the limits indicated by the author's paper for a roughness factor ratio of unity, the Kutter formula gives the following results when equated to the gravity formula: $l^{0.65} > d > l^{0.75}$. The correct exponent is determined by the specific problem.

It is interesting to compare the results obtained from the Manning and Kutter formulas with those obtained from a formula derived by Richard

Winkel, of the Technical University at Danzig. He suggests that the coefficient in the Chezy formula varies with the characteristic value, $VR\rho$ (modified Reynold's number).[10] With this as a basis, he obtained,

$$q = l\, d^{\frac{12}{7}}\, s^{\frac{4}{7}} \qquad (56)$$

in which, s = the slope ratio, model to Nature. For no artificial distortion of the slope,

$$q = l^{\frac{3}{7}}\, d^{\frac{16}{7}} \qquad (57)$$

Then, using the foregoing procedure, to satisfy the gravity and friction formulas,

$$d = l^{0.728} \qquad (58)$$

This equation gives an exponent for l that is smaller than that obtained with the Manning formula, but is within the limits indicated by the Kutter formula.

To illustrate the formulas just derived, assume a proposed model study that involves both gravity and friction. For this problem let $D_n = 40$ ft, $S_n = 0.0001$, and $N_n = 0.030$. Suppose a horizontal scale of $\frac{1}{500}$ appears suitable. According to the Manning formula, $d = l^{0.75} = \frac{1}{106}$; according to the Kutter formula, $d = l^{0.69} = \frac{1}{73}$; and according to the Winkel formula, $d = l^{0.728} = \frac{1}{92}$. Experimental data alone can provide evidence for determining which formula is the most satisfactory.

The third point to be discussed herein is the control of the roughness factor of the model. When it is assumed that N_m, the model roughness factor, and N_n, the prototype roughness factor, are equal, a determined effort should be made to produce this result. It is practically impossible to predict the roughness that will apply to a proposed model. This factor, which is actually affected by a number of items besides channel roughness, has a relatively high value in small-scale models. Probably the short radius of curvature and the small sections encountered in the model contribute most to the large values. When constructing a fixed bed model, the surface should be left as rough as practical until tests to determine the model roughness factor can be made. These tests can then be used as a basis for finishing the surface of the model to produce the desired roughness factor within the limits of its determination in the prototype.

Very little can be done in the case of a movable bed model. However, if more than one bed material is equally suitable for satisfactory bed-load movement, the material that provides the proper roughness should be selected. Usually, the sides of a movable bed model are made of concrete; that is, they are fixed, while the movable bed material is confined to the bottom of the channel. This fixed surface offers another opportunity for varying the roughness factor to some extent.

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[10] "Hydraulic Laboratory Practice," by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., 1929, p. 58.

[illegible]

[illegible] (677)

[illegible] (678)

[illegible] the Kutter formula.

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According to [illegible] according to the Kutter [illegible]

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The third [illegible] the model. When [illegible] should be [illegible] that will [illegible] This [illegible] is actually [illegible] a number of [illegible] relatively [illegible] the model [illegible] should be [illegible] the model [illegible] of the [illegible]

Very little can be done in the case of a [illegible] model. However, if [illegible]

[illegible] Laboratory Practice" by [illegible] John R. Freeman [illegible]

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

SOCIETY AFFAIRS

ANNUAL REPORT OF THE BOARD OF DIRECTION FOR THE YEAR ENDING DECEMBER 31, 1933

In compliance with the Constitution, the Board of Direction presents its Report for the year ending December 31, 1933.

THE EIGHTY-FIRST YEAR

Federal Census figures for 1930 showed 102 000 persons in the United States and its possessions to have reported themselves as Civil Engineers or Surveyors and a comprehensive analysis, made in the early part of this year by the Society's Committee on Salaries, shows the 31st of December, 1930, to have been the moment of greatest employment of civil engineers for all time in this country. That survey also shows decrease of employment of civil engineers persistently since that date and permitted the estimate that on December 31, 1933, there would be between 45 000 and 50 000 civil engineers and engineering assistants out of employment. The survey indicates, further, that by that time 28 000 would have been out of employment for a period of one year or more, and 12 000 for two years or more.

The figures are impressive but they are figures only. The individuals, members of this Society, and otherwise, concerning whom these figures were compiled, are trained men upon whom the accident of unemployment for one, two, and even three years inflicted untold hardship and privations.

It should be no occasion for wonder, therefore, that the activities of the Society during the past year turned abruptly into new lanes of endeavor. Many activities formerly valued were sharply limited in scope and both energy and money diverted to activities hitherto not fully developed or even engaged upon. It is appropriate, therefore, that the Annual Report for the year 1933 dwell particularly upon the activities newly instituted or intensified.

Public Works

In 1932 the Board of Direction urged the stimulation of public works construction as a measure capable of being of assistance in the restoration of general business. A Committee on Public Works was appointed and addressed itself actively to the passage by the Congress of the United States of measures permitting the extension of Federal credit to political subdivisions for their aid in the restoration of a normal program of public works construction. The application of Federal credit through the Reconstruction Finance Corporation was the result. This year at the instance of the Board of Direction, equally active effort was made to effect through the Congress an extension of Federal credit to a larger field of construction activities by the

political subdivisions. The authorization of $3 300 000 000 for the furtherance of public works construction through the Public Works Administration eventuated and by the end of the year, although by no means fully effective, an increase in employment of civil engineers and of general construction labor both at the site and at the place of fabrication was measurable. The Society made its biographical records fully available to the Administration during the period of the upbuilding of the Public Works Administration.

Deemed to be a worthy phase of public works, special efforts were made to secure authorization for the immediate expansion of the topographic mapping of the country, of the work of the U. S. Coast and Geodetic Survey, and of a survey of natural resources. Success followed these efforts, more effectively near the close of the year, when, under the Civil Works Administration, supervisors were selected and active field and office parties established in every State for work under the Coast and Geodetic Survey.

Codes

Under the National Recovery Administration and in accordance with the spirit of the National Industrial Recovery Act the Society joined in a determined and long-sustained effort to consolidate the several elements of the Construction Industry, design, general contracting, and sub-contracting. Under the sponsorship of the Construction League of the United States, of which the Society is a member, a general Code Committee was appointed with engineer participation and since July, 1933, has sought to determine a basic or master code. This eventually took the form of a series of chapters, of which Chapter I includes all features that can be made uniformly applicable to all the major elements of the industry, and of which subsequent chapters deal specifically with the unfair practices and the methods of administration incident to each of the major elements.

Work upon a chapter for the Engineers' Division has been continuous since July. Conferences have been held in Washington, New York, Chicago, and San Francisco, and draft after draft has been prepared. It has been an effort requiring almost infinite patience. Throughout the preparation of the Engineers' Division Code there has been a continued effort to establish an adequate definition of the engineer and the engineering assistant as technically trained and skilled men, imbued with a high appreciation of ethics and fair practices. At the time of submission of the Engineers' Division Code for Public Hearing the Code had the avowed support of four other national and eleven local or regional professional engineering societies. At the close of the year the Engineers' Code was undergoing a tentative revision looking toward a broadening of the field so as to be inclusive of engineering as it is practiced in the design and installation of special methods and processes. Two features of the Engineers' Code have called for general participation by all the members of the Society. Through the medium of the officers of the Society data with respect to fees have been submitted to a committee charged with the preparation of a proposed schedule of fees and, similarly, to all Local Sections has been assigned the collection of authoritative data on salaries to be submitted to a committee charged with the preparation of a proposed schedule of salaries.

Unemployment Relief

Again this year a strong effort has been made by the Local Sections of the Society to seek out and assist those engineers upon whom the problems of unemployment have rested heavily. Their officers have been made known to the authorities in charge of the various activities at Washington and upon more than one occasion the Local Sections, where so organized as to be effective, have been made the points of contact between these authorities and engineers who could be of assistance.

Membership

Continuing its policy of the previous year the Board of Direction has extended the privilege of suspended dues to those members who have loyally supported the Society in times past. On July 1, last, the back dues of 2 645 members, in all grades, were remitted. On October 16, last, the proposal of suspension of 1933 dues was made to 3 689 members, the requirements being a minimum period of affiliation with the Society and a simple notice to the Secretary that because of unemployment the payment of dues was impossible or impracticable. The effect of the depression upon Society members as indicated by failure to pay dues has shown an increase this year as compared with 1932, nevertheless, it is a matter for congratulation that it has not been greater. In 1932, delinquency in payment of dues was 16%; in 1933, it was 24 per cent.

Curtailments

In conformity with decreased income, expenditures were decreased for publications, for support of Local Sections, for research, administrative, professional, and technical committees. The Spring and Fall Meetings were omitted. The salaries of the staff were decreased. The intent at the time of the preparation and later modifications of the Budget was to expend a reasonable amount in excess of income. However, as a result in part of a reverse in the trend of payment of dues in the last three months of the year, the year closed with an excess of income over expenditures equal to $16 184.00, an amount, incidentally, closely approximating the sum to be paid, early in 1934, on the mortgage.

MEETINGS OF THE BOARD OF DIRECTION

There have been five meetings of the Board of Direction during 1933:

January 16–17, New York, N. Y.
January 19, New York, N. Y.
May 12, New York, N. Y.
June 25–26, Chicago, Ill.
September 25–26, Chicago, Ill.

There have been five meetings of the Executive Committee:

May 12, New York, N. Y.
June 26, Chicago, Ill.
August 25–26, New York, N. Y
September 26, Chicago, Ill.
December 15, New York, N. Y.

MEMBERSHIP

The changes in membership are shown in the following table:

	Jan. 1, 1933			Jan. 1, 1934			Losses				Additions			Totals		
	Resident	Non-Resident	Total	Resident	Non-Resident	Total	Transfer	Resignation	Dropped	Died	Transfer	Election	Reinstatement	Loss	Gain	Increase
Honorary Members	5	14	19	5	13	18	...	...	...	2	*1	...	...	2	1	§1
Members	992	4 829	5 821	964	4 789	5 753	1	33	14	122	†68	30	4	170	102	§68
Associate Members	1 090	5 241	6 331	1 050	5 220	6 270	68	65	68	37	‡57	115	5	238	177	§61
Juniors	549	2 407	2 956	547	2 499	3 046	57	63	¶164	5	0	378	1	289	379	90
Affiliates	37	78	115	34	75	109	0	3	1	3	0	0	1	7	1	§6
Fellows	3	2	5	3	2	5	...	...	...	...	...	...	...	...	...	...
Total	2 676	12 571	15 247	2 603	12 598	15 201	126	164	247	169	126	523	11	706	660	§46

* 1 Member.
† 68 Associate Members.
‡ 57 Juniors.
§ Decrease.
¶ 82 Juniors dropped on account of age limit.

New Members and Net Increase

The following table shows the new members and the net increase during the past ten years. The diagram on page 5 gives membership statistics for the same period:

	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933
New Members*	715	795	1 072	1 139	1 244	1 066	1 139	1 055	753	534
Net Increase	262	111	721	755	820	508	574	531	57	46§

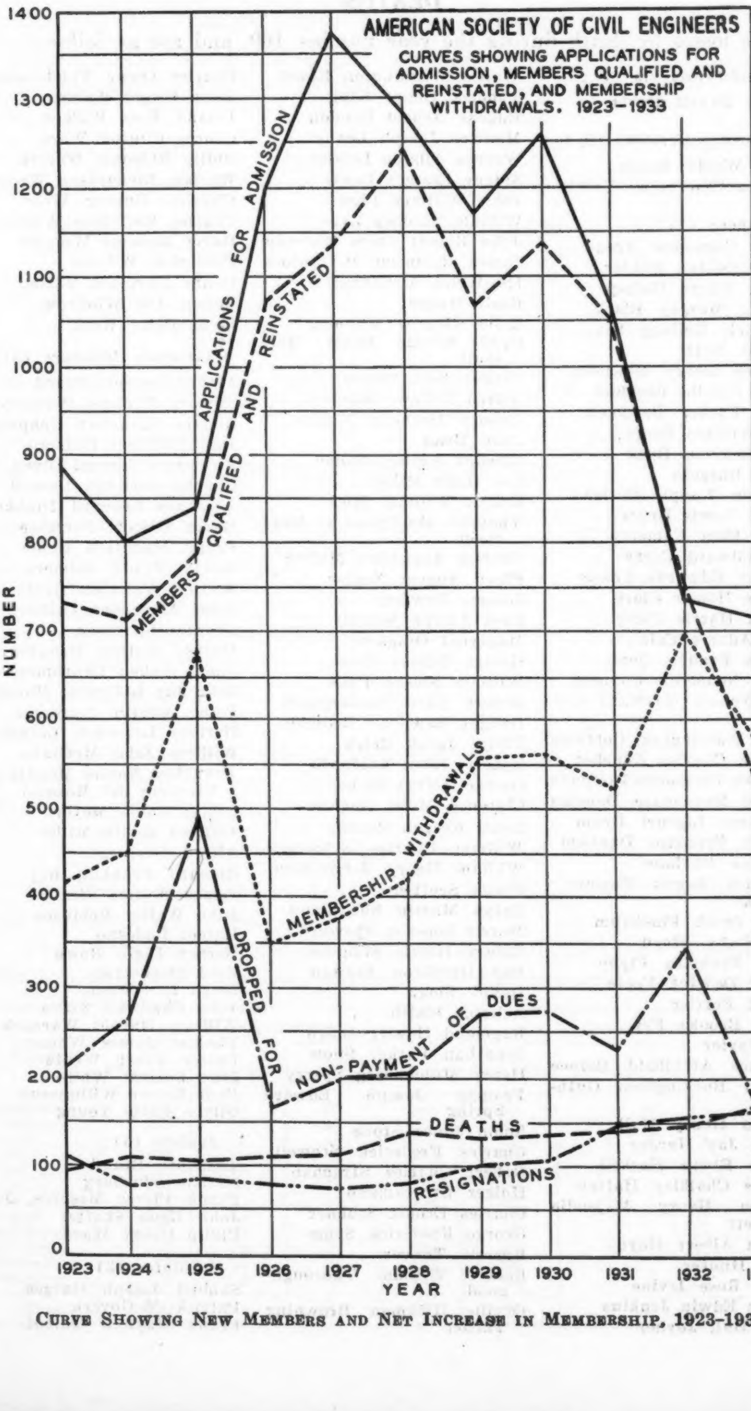
* Includes reinstatements.
§ Decrease.

Applications for Membership

The total number of applications for membership was 887, of which 715 were for admission and 172 for transfer.

The number of applications received during the past ten years follows:

	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933
For admission	800	848	1 194	1 374	1 284	1 172	1 260	1 072	736	715
For transfer	239	256	292	304	274	271	338	224	192	172
Total	1 039	1 104	1 486	1 678	1 558	1 443	1 598	1 296	928	887



CURVE SHOWING NEW MEMBERS AND NET INCREASE IN MEMBERSHIP, 1923–1933

DEATHS

The losses by death during the year number 169, and are as follows:

Past-Presidents (1):

Arthur Powell Davis

Honorary Members (2):

Jonas Waldo Smith
William Cawthorne Unwin

Members (121):

Albert Cornelius Arend
John Crichton Baxter
Lyman Edgar Bishop
William Murray Black
Frederick Kellogg Blue
John B. Bott
Clarence Henry Bowman
Henry Cyrille Breidert
James Parker Brownell
Fred William Bruce
Con Morrison Buck
Harry Burgess
Matthew Joseph Butler
Morton Lewis Byers
Harry Ezra Cameron
John Edward Carty
Clement Edwards Chase
Charles Homer Clark
Branch Harris Colby
John Adams Cole
Charles Francis Conn
Albert Neumann Connett
John Nelson Hayward Cornell
Lewis Warrington Cottman
Maurice Charles Couchot
Abraham Fairbanks Doremus
Edward Morehouse Douglas
Alphonsus Ligouri Drum
Herbert Franklin Dunham
Leicester Durham
Frederick James Easterbrook
Harry Jacob Finebaum
Earl Nesby Floyd
Harry Franklin Flynn
Arthur DeWint Foote
Samuel Fortier
Alfred Brooks Fry
Carl Gayler
Laurence Archibald Goines
Edward Buckingham Guthrie
William Henry Hall
Harold Jay Harder
Eugene Elwin Haskell
Thomas Chalkley Hatton
Bertram Henry Majendie Hewett
Warren Albert Hoyt
Adam Hunter
Vivion Rose Irvine
Charles Edwin Jenkins
Frank Hall Joyner
Walter Washington Kerch
Thomas Henry King
Eugene Ashbel Landon
Maurice Joseph Leahy
Warren Allston Leland
Sidney Francis Lewis
John William Link
William Thomas Lyle
John Robert Clark Macredie
James Robinson McClintock
Ira Welch McConnell
Basil Magor
Rollo Glenroy Manning
Cyril Ernest Davis Marshall
Arthur John Mason
Alfred Fellows Masury
Dabney Herndon Maury
John Mead
Edward Furber Miller
Lee Haun Miller
Ernest William Moir
Thomas Montgomery Morrison
Charles Augustine Mullen
Floyd August Nagler
Jerome Newman
Fred Adolph Noetzli
Masayuki Otagawa
George Ellison Otis
William Nelson Page
Robert Allen Pendergrass
Dwight Raymond Redman
Philip Jacob Reich
Robert John Reidpath
George Alfred Ricker
Clarence Mord Rogers
Louis Keegan Rourke
William Charles Schnabel
William Henry Schuerman
Frank Scotten
Ralph Martin Shankland
Morris Robeson Sherrerd
Robert Harris Simpson
Ray Hamilton Skelton
David Sloan
Acheson Smith
Raymond Hewitt Smith
Jonathan Parker Snow
Henry Muhlenberg Sperry
Francis Joseph Edward Spring
Solon Jones Stone
Charles Frederick Stowell
Robert Charles Strachan
Holger Struckmann
Charles Ernest Stubner
George Frederick Syme
Russell Thayer
Robert William Thoroughgood
Orville Hickman Browning Turner
Charles Oscar Vandevanter
Jesse Wager Walker
Leland Ross Walker
George Clinton Ward
Philip Ridsdale Warren
William Richardson Webster
Charles Hunter West
Charles Valentine Weston
Harry Roberts Wheeler
Frederick Wilcock
Henry Harrison Wilson
Rollen Joe Windrow
Louis Peter Wolff

Associate Members (37):

Arthur Garfield Beard
Edward Michael Brennan
Ronald Farquhar Chapman
Daniel Walter Dillman
Alexander Samuel Diven, 3d
Charles Steward Donald
Abraham Leonard Drabkin
Harry Hilbert Ferrebee
Frank Hamilton Finch
David Pryde Gilmore
Andrew Valerian Greaves
John Alexander Griffin
George Halverson
Harvey Sydney Henning
Simon Cohen Henriques
John Jay Lafayette Houston
Peter Martin Louwerse
Herbert Lawrence Luther
William John McGrath
Francisco Xavier Memije y Vivencio del Rosario
Lester Louis Meyer
Clifford Neville Miller
Alfred Norberg
Richard Franklin Rey
James Horner Rice
John Walter Robinson
Rafael Rodezno
Berney Elgin Rowe
Paul Steenstrup
Leigh E. Stevens
John Chadwick Stiles
William Harold Warnock
Thomas James Wasser
Foster Pratt Wentz
Ivan Forrest White
Paul Revere Williamson
Oliver Earle Young

Juniors (5):

Theodore Brand
Nathan Ginsburg
Frank Pierce Meserve, Jr.
John Egon Skafte
Philip Henry Ward

Affiliates (3)

Samuel Joseph Garges
Patrick McGovern
Frank Mayhew Talbot

ENGINEERING SOCIETIES LIBRARY

The statistics which follow give comparative figures for 1932 and 1933 of the Engineering Societies Library:

	1932		1933	
Additions:				
Volumes (by gift)	2 452		1 789	
" (by purchase)	1 212	3 672	1 124	2 913
Pamphlets (by gift)	3 540		3 339	
" (by purchase)	322	3 862	272	3 611
Maps (by gift)	256		141	
" (by purchase)	17	273	8	149
Searches		31		26
Total additions		7 830		6 699
Permanent collection		144 648		143 660
Expenditures for books, periodicals, binding, supplies, and salaries (approximate)		$48 197		$40 077
The Library was used by		45 435		42 915
Including personal visits by		33 882		33 258
Volumes catalogued		4 179		3 360
Cards added to catalog		26 483		17 237
Total catalog cards, arranged under subject		477 422		465 016
Searches made		70		49
Translations made		138		97
Photoprints made		24 675		16 889
Number of persons securing photographs		3 041		2 215
Receipts for service		$9 481		$8 904
Members borrowing books		207		111

EMPLOYMENT SERVICE

The Employment Service has offices in New York, N. Y., Chicago, Ill., and San Francisco, Calif.

The number of men placed during 1933 has averaged about 72 per month. The following table shows the registrations and placements in the three offices:

Month	Men Registered				Men Placed			
	New York	Chicago	San Francisco	Total	New York	Chicago	San Francisco	Total
January	148	55	65	268	40	9	10	59
February	109	52	46	207	27	7	5	39
March	98	41	48	187	37	6	9	52
April	107	38	42	187	30	8	6	44
May	122	46	66	134	35	35	28	98
June	156	33	52	241	50	12	25	87
July	128	51	57	236	33	9	22	64
August	144	45	74	263	42	11	21	74
September	126	43	46	215	48	4	10	62
October	100	24	31	155	62	8	21	91
November	77	17	26	120	63	11	44	118
December	75	4	38	117	52	5	25	82
Total	1 390	449	591	2 430	519	125	226	870

PUBLICATIONS

In 1933, the Society has published two volumes of *Transactions* (Volumes 97 and 98), ten numbers of *Proceedings,* twelve numbers of *Civil Engineering,* and a Year Book.

Transactions.—Of the two volumes of *Transactions,* Volume 97 is made up of the papers and discussions describing the various design and construction problems of the George Washington Bridge, written by the Engineers of the Port of New York Authority engaged on the work. Volume 98 is the regular yearly issue of *Transactions* and contains the papers and discussions published in *Proceedings* from August, 1931, through May, 1932, as well as the Annual Address, by President Alonzo J. Hammond, the Final Report of the Special Committee on Steel Column Research, and the Memoirs of Deceased Members. Volumes 97 and 98 of *Transactions* were issued as Parts 2 and 3 of October, 1933, *Proceedings.*

Proceedings.—During the year Progress Reports of the Special Committee on Meteorological Data and the Special Committee on Earths and Foundations, and the Third Progress Report of Sub-Committee No. 31, Committee on Steel, of the Structural Division, on Wind-Bracing in Steel Buildings were published in January, May, and December *Proceedings* respectively. There have also been published in *Proceedings* 30 papers and 1 Symposium, together with the discussions thereon, and also a number of discussions of papers published in 1932. Of the papers published in 1933, there were included four of the series of eight papers published jointly by the Port of New York Authority and the Society. These papers appeared in the January and February, 1933, *Proceedings,* respectively.

The number of members and others taking part in the preparation and discussion of these papers, symposium, reports, and discussions thereon, was 119. Discussions of papers, etc., published in *Proceedings* for 1931 and 1932, to the number of 121 were also included in the 1933 issues, making a total of 240 authors and discussers.

Civil Engineering.—Special issues of *Civil Engineering* for the year included the March number containing abstracts of the papers delivered before the Annual Meeting of the Society in January, 1933; and the August and September numbers covering abstracts from the Chicago Convention in June, 1933. The remainder were regular issues. In all, 144 papers appeared during the year, including the abstracts. In addition, 21 brief papers under the heading, "Engineers' Notebook," were printed. Discussions and comments to the number of 129 contributions were scattered throughout the twelve issues. News and records of Society affairs, membership data, reviews of current periodical literature, and items of interest to civil engineers were also included in each issue. The program for the Annual Meeting was issued complete in the January number and that for the June Convention, in the June number.

Memoirs.—In October, 1930, the publication of memoirs of deceased members was discontinued in *Proceedings* and a pamphlet form of memoir was adopted, with final publication in *Transactions.* Since that time approximately 400 memoirs have been issued many of which have been included in *Transactions* Vol. 95 (1931), Vol. 96 (1932), and Vol. 98 (1933).

Stock of Publications.—The stock of the various publications of the Society kept on hand for the convenience of members and others now amounts to 186 308 copies, the cost of which to the Society for paper and press work only has been $32 369.49.

Cost of Publications, Etc.—The table (see page 10) shows the cost per page for text and illustrations in *Proceedings and Transactions* for the past sixteen years and in *Civil Engineering* for the past four years.

The various topics developed in *Transactions, Proceedings,* and *Civil Engineering* during the year and the number of pages devoted to each are, as follows:

Subject	*Transactions,* pages	*Proceedings,* pages	*Civil Engineering,* pages
City and Regional Planning		15	25
Construction Equipment			5
Contracts	7	7	
Dams	236	301	38
Drainage and Irrigation	15	72	5
Earthquakes	55		
Engineering Economics			28
Engineering History	8	58	4
Erosion			5
Excavation			1
Fishways			5
Floods			9
Forestation		49	4
Foundations		44	25
Highway Engineering	135	46	33
Hydrology, Hydraulics		137	18
Materials of Engineering	60	43	13
Mathematics			2
Meteorology		92	3
Military Engineering			5
Power Plants	33	8	
Power Transmission	27		
Quarrying			3
Railways		29	16
Sanitation			21
Sewage Disposal		36	11
Soils	91	120	3
Sound Control			2
Structural Engineering	1 115	484	94
Surveying	56	32	17
Transportation		27	18
Tunnels			4
Water Supply			25
Water Works		73	
Waterways	74	11	27
Letters to the Editor			55
Memoirs	227		
Society Affairs		26	101
Items of Interest			27
Local Sections			12
Student Chapters			10
Current Periodical Literature			36
Recent Books			7
Men and Positions Available			14
Changes in Membership Grades			11
News of Engineers			7
	2 139	1 710	749

TABLE SHOWING NUMBER AND COST OF PAGES AND COST OF ILLUSTRATIONS FOR *Transactions, Proceedings,* AND *Civil Engineering.*

Year	Issues	Edition	Pages: Per volume	Pages: Total	Issues	Edition	Pages: Per volume	Pages: Total	Total pages	Total cost	Cost per page	Cost	Percentage of total cost	Cost per page
	TRANSACTIONS				*PROCEEDINGS,* PART I*				*PROCEEDINGS* (PART I)* and *TRANSACTIONS*			ILLUSTRATIONS		
1918	1	8 700	1 879	16 340 000	10	8 950	2 341	20 950 000	37 290 000	33 785.64	0.00091	1 192.20	3.5	0.000032
1919	1	9 000	1 775	15 980 000	8	9 100	2 096	19 075 000	35 055 000	32 082.69	0.00091	1 128.53	3.5	0.000032
1920	..	......	.....	....	10	10 142	2 014	20 440 000	20 440 000	23 446.34	0.00115	2 552.27	10.9	0.000125
1921	2	10 000 10 500	2 479 993	35 212 000	10	10 680	1 834	19 450 000	54 662 000	66 298.39	0.00121	2 034.72	3.1	0.000037
1922	1	10 900	1 826	19 900 000	10	11 100	2 740	30 400 000	50 300 000	56 200.00	0.00112	3 700.00	6.6	0.000073
1923	1	11 200	1 808	20 250 000	10	11 500	3 210	36 915 000	57 165 000	60 612.83	0.00106	4 809.88	7.9	0.000084
1924	1	11 500	1 515	17 440 000	10	11 750	2 612	30 700 000	48 140 000	47 573.72	0.00099	4 579.04	9.6	0.000095
1925	1	11 400	1 538	17 533 000	10	11 800	2 910	34 338 000	51 871 000	47 409.07	0.00091	6 359.19	13.4	0.000122
1926	..	12 100	1 786	21 610 000	10	12 200	3 046	37 161 000	58 771 000	53 473.54	0.00091	5 080.91	9.5	0.000087
1927	2	12 900 13 200	1 234 1 229	15 919 000 16 223 000	10	13 050	3 720	48 546 000	80 688 000	65 972.76	0.00080	9 279.49	14.0	0.000126
1928	1	14 000	1 860	26 040 000	10	14 200	4 016	57 027 000	83 067 000	61 587.98	0.00074	7 249.00	12.4	0.000087
1929	1	14 500	2 030	29 435 000	10	14 540	3 566	51 850 000	81 285 000	58 388.39	0.00072	4 011.95	6.9	0.000049
1930	1	15 100	1 794	27 090 000	10	15 170	2 936	44 540 000	71 630 000	52 881.95	0.00074	4 189.56	7.9	0.000058
1931	1	15 500	1 720	26 660 000	10	15 820	1 968	31 133 000	57 793 000	43 579.42	0.00075	3 680.95	8.5	0.000064
1932	1	15 800	1 688	26 670 000	10	15 700	2 160	34 010 000	60 680 000	37 348.87	0.00062	4 830.47	12.9	0.000080
1933	2	12 000 12 700	448 1 752	5 378 000 22 303 000	10	13 520	1 784	24 075 000	51 756 000	32 402.35	0.00063	3 653.66	11.3	0.000071
	CIVIL ENGINEERING													
1930	..	......	.....	....	3	16 230	248	4 026 000	4 026 000	8 969.66	0.00223	995.06	10.1	0.000247
1931	..	......	.....	....	12	16 275	1 264	20 572 000	20 572 000	41 012.75	0.00199	6 271.40	15.3	0.000305
1932	..	......	.....	....	12	16 020	1 048	16 786 000	16 786 000	33 057.95	0.00197	5 112.68	15.4	0.000305
1933	..	......	.....	....	12	13 500	912	12 316 000	12 316 000	23 678.54	0.00192	3 762.72	15.9	0.000305

* Includes Part III, May 1928, 288 pp.; Part 2, May 1932, 112 pp.

Summary of Publications for 1933

	Issues	Average edition	Total pages	Cuts
Proceedings (monthly numbers)..........	10	14 060	1 784	501
Civil Engineering (monthly numbers)......	12	13 504	864	775
Transactions Vol. 97......................	1	12 100	446	180
Transactions Vol. 98......................	1	12 700	1 746	590
Year Book	1	15 850	504	4
Total	25	.....	5 344	2 050

The gross cost of publications, as determined by the bills actually paid during the year, and inclusive of salaries, has been:

Technical Publications	$101 518.07
General Publications	5 580.89
Total	$107 098.96

READING ROOM OF THE SOCIETY

The attendance at the Reading Room during the year was 4 014.

Two hundred and sixty periodicals are regularly received. Included in this number are many foreign periodicals, also a number of literary magazines and several daily newspapers.

MEETINGS

Six meetings of 6 sessions were held during the year as follows: At the Annual Meeting at New York, N. Y., 2 (2 sessions); at the Annual Convention, at Chicago, Ill., 2 (2 sessions); and 2 regular meetings held in the Engineering Societies Building, New York, N. Y.

At these meetings there were presented two Symposia, six Reports of Special Committees of the Society, and four Addresses.

The total attendance at the meetings of the Society during the year was approximately 2 750. The registered attendance at the Annual Meeting was 1 681, and at the Annual Convention, 1 029.

The dates of the meetings of the Society during the year, together with the titles of the Symposia, Reports, Addresses, etc., presented thereat, are as follows:

January 18, 1933 (Two Sessions): Reports of Special Committees on Concrete and Reinforced Concrete, Earths and Foundations, Irrigation Hydraulics, Meteorological Data, Steel Column Research, and Stresses in Railroad Track; and Symposium on "Long Range Planning, The Future of Business, International Relations, and Railway Problems."[1]

March 15, 1933 (One Session): Business Meeting of Society.

June 27, 1933 (Two Sessions): "A Century of Civil Engineering in the United States," Address by Alonzo J. Hammond, President, Am. Soc. C. E.[2]; and Symposium on "Tax Reduction."[3]

October 18, 1933 (One Session): Business Meeting of the Society.

[1] *Civil Engineering*, March, 1933, pp. 119, 134, 137.
[2] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1463.
[3] *Civil Engineering*, August, 1933, p. 417 *et seq.*

MEDALS, PRIZES, AND AWARDS

The award of Medals and Prizes for the year ending July, 1933, was as follows:

The Norman Medal to Hardy Cross, M. Am. Soc. C. E., for his paper entitled "Analysis of Continuous Frames by Distributing Fixed-End Moments."

The J. James R. Croes Medal to Earl I. Brown, M. Am. Soc. C. E., for his paper entitled "Flow of Water in Tidal Canals."

The Thomas Fitch Rowland Prize to John C. Baxter, M. Am. Soc. C. E., for his paper entitled "The Eight-Mile Cascade Tunnel, Great Northern Railway: Construction Plans and Methods."

The James Laurie Prize to Walter B. Saunders, M. Am. Soc. C. E., for his paper entitled "Construction of La Ola Pipe Line, in Chile."

The Arthur M. Wellington Prize to Duncan J. Kerr, M. Am. Soc. C. E., for his paper entitled "The Eight-Mile Cascade Tunnel, Great Northern Railway: Preliminary Studies and Results of Improving Cascade Crossing."

The Collingwood Prize for Juniors to Bernard L. Weiner, Assoc. M. Am. Soc. C. E., for his paper entitled "Design of a Reinforced Concrete Skew Arch."

The third award of the Alfred Noble Prize was made to C. Maxwell Stanley, Jun. Am. Soc. C. E., for his paper "Study of Stilling Basin Design", published in *Proceedings* for November, 1932.

LOCAL SECTIONS

The number of Local Sections, 56, is unchanged since last year, no Section having been organized or disbanded during 1933.

TECHNICAL DIVISIONS

All but one of the Technical Divisions of the Society held sessions during the year, either at the Annual Meeting in New York in January, or at the Annual Convention in Chicago in June. Of these meetings, five were double sessions, of which four were held jointly with similar Divisions of other Societies at the Annual Convention during Engineers' Week of the Century of Progress.

The meetings of the Divisions were marked by good attendance and interest and by an excellent group of technical papers. Abstracts of all the presentations appeared, either in the March or the August and September issues of *Civil Engineering*. In addition, some of the papers are being prepared for publication in more complete form in *Proceedings* and *Transactions*. The various papers are listed chronologically under the Divisions which sponsored the meetings.

City Planning Division

January 19, 1933, "Equitable Distribution of Assessments for City Planning Projects", by Hyman Shifrin, M. Am. Soc. C. E.

June 30, 1933, "Preparedness for Slum Clearance", by John H. Millar, Esq.; "Value of Planned City Development", by Hugh E. Young, M. Am. Soc. C. E.; "State and Regional Planning", by Jacob L. Crane, Jr., M. Am. Soc. C. E.; and "New Opportunities for the Engineer in City Planning", by D. H. Sawyer, M. Am. Soc. C. E.

Construction Division

January 19, 1933 (Joint Session with Structural Division), "The New Jersey Express Highway", by H. W. Hudson, M. Am. Soc. C. E.; "Construction of Railroad Grade Separation", by G. H. Wilsey, M. Am. Soc. C. E.; and "Structural Design for Railroad Grade Separations", by John L. Vogel, M. Am. Soc. C. E. ,

June 29, 1933, "The Construction Engineer—The Centenarian", by W. C. Huntington, M. Am. Soc C. E.; "A Century of Progress in the Construction of Transportation Facilities", by F. G. Jonah, Vice-President, Am. Soc. C. E.; "A Century of Progress in Water Transportation", by J. Howland Gardner, President, Society of Naval Architects and Marine Engineers; "A Century of Progress in the Development of Bridge Construction", by O. H. Ammann, M. Am. Soc. C. E.; "A Century of Progress in Methods of Constructing Municipal Facilities", by Samuel A. Greeley, M. Am. Soc. C. E.; "A Century of Progress in Evolution of Methods in the Construction of Reclamation Projects", by Elwood Mead, M. Am. Soc. C. E.; "A Century of Progress in Methods of Construction of National Defense Facilities on Water", by R. E. Bakenhus, M. Am. Soc. C. E.; "National Defense Facilities on Land", by Lytle Brown, M. Am. Soc. C. E.; "Review of Methods in the Construction of Water Terminals Over the Past Century", by Walter J. Cahill, M. Am. Soc. C. E.; "A Century of Construction Machinery", by F. C. Ruhloff, Esq.; "A Century of Progress in the Construction of Industrial Buildings", by T. L. Condron, M. Am. Soc. C. E.; and "Sound Control in the Buildings of the Future", by Paul E. Sabine, Esq.

Highway Division

January 19, 1933, "Cost of Delays", by S. Johannesson, M. Am. Soc. C. E.; "Control of Pavement Cuts with Reference to Washington Practice", by H. C. Whitehurst, M. Am. Soc. C. E.; and "Control of Pavement Cuts with Particular Reference to New York City Practice", by R. A. MacGregor, M. Am. Soc. C. E.

June 29, 1933, "Highway Improvement—How to Keep It Sold to the Public", by E. J. Mehren, M. Am. Soc. C. E.; and "The Time Element in Highway Traffic Movement", by E. W. James, M. Am. Soc. C. E., and Miller McClintock, Esq.

Power Division

June 29, 1933 (Jointly with Hydraulics Division of American Society of Mechanical Engineers), "Flow of Water Around Bends in Open and Closed Channels", by David L. Yarnell and Floyd A. Nagler, Members, Am.

Soc. C. E.; Report of Power Division's Committee on Legislation Respecting Safety of Dams, by R. A. Monroe, M. Am. Soc. C. E.; and "An Analysis of the Proposed Official Plans for Water Power Development on the International Section of the St. Lawrence River"[4], by Daniel W. Mead, Hon. M. Am. Soc. C. E.

June 30, 1933 (Jointly with Hydraulics Division of American Society of Mechanical Engineers), Committee Report Reviewing the Present Status of Water Hammer Theory; "A Simplified Derivation of Water Hammer Formula", by Lewis F. Moody, Esq.; "The Effect of Surge Tanks and Surge Tank Risers on Water Hammer Computations", by Eugene E. Halmos, M. Am. Soc. C. E.; "High Head Penstock Design", by A. W. K. Billings, M. Am. Soc. C. E., and Messrs. O. H. Dodkin and F. Knapp, and Adolpho Santos, Jr, Jun. Am. Soc. C. E.; "Influence of Water Hammer on Design of High-Head Penstocks at the Drum Plant and the Tiger Creek Plant", by Walter Dreyer, M. Am. Soc. C. E.; "Computation of Water Hammer Pressures in Compound Pipes", R. E. Glover, Esq.; "Surge Control in Centrifugal Pump Discharge Lines", by Ray S. Quick, M. Am. Soc. C. E.; and "Water Hammer Tests in Croton Lake Pumping Plant", by S. Logan Kerr, Assoc. M. Am. Soc. C. E.

Sanitary Engineering Division

January 19, 1933, "The Manufacture and Control of Liquid Alum at Montebello Filters", by J W. Armstrong, M. Am. Soc. C. E.; "Deep Tunnel Delivery of Water Supply for Large Cities", by Walter E. Spear, M. Am. Soc. C. E.; "Formation of Floc with Ferric Salts", by Edward Bartow, M. Am. Soc. C. E., A. P. Black, Esq., and Walter E. Sansbury, Esq.; and "Status of Wards Island Sewage Treatment Works", by Richard H. Gould, M. Am. Soc. C. E.

June 30, 1933, "Present Status of Sewage Treatment at Chicago, and Other Sewage Treatment Projects in Relation to the Illinois River", by Langdon Pearse, M. Am. Soc C. E.; "Stream Cleansing—The Sangamon River", by W. D. Hatfield, Assoc. M. Am. Soc. C. E.; "Stream Cleansing", by Almon L. Fales, M. Am. Soc. C E.; "The Problem of Sewage Treatment at Duluth", by John Wilson, M. Am. Soc. C. E.; "Industrial Wastes, Their Relative Importance in Stream Pollution", by L. F. Warrick, Esq.; and "Pollution of the Southern End of Lake Michigan", by Arthur E. Gorman, Esq., and John R Baylis, Assoc. M. Am. Soc. C. E.

Structural Division

January 19, 1933, Report of Sub-Committee on Wind Bracing in Tall Buildings", by C. R. Young, M. Am. Soc. C. E.; Report of Sub-Committee on Structural Alloy and Heat Treated Steels", by Robert S. Johnston, M. Am. Soc. C. E.; and "Wind Stress Analysis Simplified", by L. E. Grinter, Assoc. M. Am. Soc. C. E.

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[4] Also jointly with American Institute of Electrical Engineers.

June 29, 1933 (Jointly with Applied Mechanics Division of American Society of Mechanical Engineers), "The Rational Design of Steel Columns", by D. H. Young, Jun. Am. Soc. C. E.; "Stability of the Web of Plate Girders", by S. Timoshenko, Esq.; "Stability of Thin Walled Tubes Under Torsion", by L. H. Donnell, Esq.; "Laboratory Tests of Multiple-Span Reinforced Concrete Arches", by W. M. Wilson, M. Am. Soc. C. E.; "Wind Pressure on Buildings," by Dr. Eng. O. Flachsbart; and "Tests of Split-H End Connections for Wind Girders", by W. C. Huntington, M. Am. Soc. C. E.

June 30, 1933 (Jointly with Applied Mechanics Division of American Society of Mechanical Engineers), "Impact Effect on Bridges", by Dr. R. Bernhard; "Graphostatics of Stress Functions", by H. M. Westergaard, M. Am. Soc. C. E.; "The Amplitudes of Non-Harmonic Vibrations", by J. P. Den Hartog, Esq.; "A Generalized Deflection Theory for Suspension Bridges Including the Analysis of Continuous Spans", by D. B. Steinman, M. Am. Soc. C. E.; "A Suspension Stiffening Truss of Tension Members as Developed for the Chicago Skyway", by William G. Grove, M. Am. Soc. C. E.; and "The Theory of the Suspension Bridge", by A. A. Jakkula, Jun. Am. Soc. C. E.

Surveying and Mapping Division

June 29, 1933, "City Survey.—Past, Present and Future", by G. D. Whitmore, Assoc. M. Am. Soc. C. E.; and "One Hundred Years of Control Surveys", by William Bowie, M. Am. Soc. C. E.

June 30, 1933, "Land Surveying—Its Foundation and Superstructure", by M. L. Greeley, Esq.; and "The Land Surveyor's Starting Point", by W. D. Jones, Esq.

Waterways Division

January 19, 1933, "What Are Navigable Waters of the United States?", by G. B. Pillsbury, M. Am. Soc. C. E.

June 29, 1933, "Chicago Terminus of the Lakes-to-Gulf Waterway", by Daniel I. Sultan, Esq.

MEMBERSHIP OF TECHNICAL DIVISIONS

City Planning	1 532
Construction	2 671
Engineering-Economics and Finance	457
Highway	2 190
Irrigation	941
Power	870
Sanitary Engineering	1 640
Structural	2 789
Surveying and Mapping	895
Waterways	872
Total	14 857

STUDENT CHAPTERS

There are at present 109 Student Chapters. The following were organized during 1933:

D. B. Jett (New Mexico State College)
Duke University
Lewis Institute
South Dakota State College
Texas Technological College
Tulane University

The reports of the Secretary and Treasurer are appended.

By order of the Board of Direction,

GEORGE T. SEABURY,
Secretary.

January 15, 1934.

REPORT OF THE TREASURER OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS FOR THE YEAR ENDING DECEMBER 31, 1933

In compliance with the provisions of the Constitution, I have the honor to present the following report:

Cash on hand January 1, 1933		$40 515.31
Receipts		
From Current Sources, January 1 to December 31, 1933	$266 617.59	
Rent from 57th Street Property	52 500.00	
Chicago, Ill., Tax Warrant (Called)	1 006.30	
Interest on Investments	865.86	320 989.75
Disbursements		
Payment of Bills by Audited Vouchers, January 1 to December 31, 1933	$303 279.24	
Payment on Mortgage	12 000.00	
Cash on hand December 31, 1933	46 225.82	
	$361 505.06	$361 505.06

Respectfully submitted,

OTIS E. HOVEY,

Treasurer.

BALANCE SHEET AT

ASSETS

Cash:			
In banks and on hand................	$17 113.76		
On deposit with U. S. Post Office......	200.00	$17 313.76	
Marketable securities at cost and accrued interest ($21 380 at market quotations and accrued interest).....	....	29 148.26	
Accounts receivable:			
Members	78 148.09		
Non-members........................	3 134.80		
	81 282.89		
Less, allowance for doubtful accounts.	41 374.00	39 908.89	
Inventory of publications on hand, at cost or less......		10 369.49	
Prepaid insurance premiums		289.87	
		97 030.27	
Real Estate:			
Interest in real estate and other assets of United Engineering Society, exclusive of trust funds..............	493 352.60		
218–220 West 57th Street, New York, N. Y., at book amount, less depreciation	602 479.44	1 095 832.04	
Furniture and office equipment, less reserve for depreciation		5 399.57	
Library (at book amount):			
Cash expended for books, etc..........	22 122.22		
Donations	72 310.83	94 433.05	$1 292 694.93
Fund Investments:			
The Fifty-seventh Street Property Fund:			
Marketable securities, at cost ($46 335 at market quotations)....	62 654.24		
Accrued interest	1 090.11		
Uninvested cash	22 966.32	86 710.67	
The Freeman Fund:			
Marketable securities, at cost ($14 591 at market quotations)............	21 207.12		
Uninvested cash	242.99	21 450.11	
Merritt Haviland Smith Memorial Fund............		1 195.16	
Rudolph Hering Medal Fund........................		531.03	109 886.97
Cash in banks, representing unexpended balances of cash received for special purposes.................		4 176.56	
Freeman Fund income and expenses..................		33.02	4 209.58
			$1 406 791.48

To THE BOARD OF DIRECTION,

AMERICAN SOCIETY OF CIVIL ENGINEERS,

We have examined the accounts of AMERICAN SOCIETY OF CIVIL ENGINEERS as to the reasonableness of the amounts at which real estate and library are included financial condition of the Society at that date.

New York, January 11, 1934.

DECEMBER 31, 1933

Liabilities and Funds

Mortgage payable, due February 1, 1935..	$18 000.00		
Interest accrued on mortgage............	375.00		
1934 membership dues paid in advance....	37 151.21		
Other member and non-member credits...	4 334.00	$59 860.21	
Prizes, library and compounded dues fund	23 852.50		
Alfred Noble Fund	16 342.55	40 195 05	
Current fund surplus, including amount arising from revaluation of real estate.............		1 192 639.67	$1 292 694.93
The Fifty-seventh Street Property Fund..............		86 710.67	
The Freeman Fund		21 450.11	
Merritt Haviland Smith Memorial Fund..............		1 195.16	
Rudolph Hering Medal Fund........................		531.03	109 886.97
Unexpended balances of cash received for special purposes:			
Special Committee on Stresses in Railroad Track....		60.28	
Special Committee on Earths and Foundations......		1 141.00	
Power Division		2 300.85	
City Planning Division		562.45	
Surveying and Mapping Division....................		90.00	
Alfred Noble Fund, not released....................		55.00	4 209.58
			$1 406 791.48

at December 31, 1933, and upon the basis of carrying securities at cost, and subject therein, we certify that, in our opinion, the above balance sheet sets forth the

Lybrand, Ross Bros. & Montgomery

REPORT OF SECRETARY FOR THE

TO THE BOARD OF DIRECTION OF THE

GENTLEMEN:—I have the honor to present a statement of Receipts and There is also appended a general Balance Sheet showing the condition of the

RECEIPTS

Cash on hand January 1, 1933			$40 515.31*
Entrance Fees		$8 790.00	
Current Dues		153 550.70	
Past Dues		6 875.34	
Advance Dues		37 151.21	
Sale of Publications		8 873.53	
Binding for Members		8 853.00	
Badges		2 745.00	
Certificates of Membership		227.50	
Annual Meeting		2 272.00	
Interest on Deposits		129.47	
Interest on Investments		865.86	
Maturity of Bonds (Chicago 29, Tax Warrants)		1 006.30	
Interest Accrued		27.00	
Postage		358.93	
Advertising		12 749.23	
Miscellaneous		1 090.27	
Special Publications		5 582.50	
Income from 57th St. Property:			
Credited to General Receipts	$44 000		
Credited to 57th St. Property Funds	8 500	52 500.00	
From Engineering Foundation in credit to:			
Special Committee on Earths and Foundations	$600		
Special Committee on Concrete and Reinforced Concrete Arches	2 000		
Technical Publications for publishing Report of Special Committee on Earths and Foundations	1 400		
Report of Special Committee on Steel Column Research	1 320	5 320.00	
The 57th St. Property Fund:			
Interest on Securities		2 542.50	
The Freeman Fund:			
Income		1 224.76	
Principal (Sale of Securities)		2 826.59	
The Alfred Noble Fund:			
Interest on Invested Funds		275.00	
Interest on not Released Funds		3.30	
City Planning Division:			
Dues		10.00	
Interest		16.04	
Surveying and Mapping Division:			
Dues		5.00	
Power Division:			
Interest		67.85	
Rudolph Hering Medal Fund:			
Interest		15.70	
Merritt H. Smith Memorial:			
Interest		35.17	
Repayment of U. E. T. Loan		5 000.00	320 989.75
			$361 505.06

* For itemization see page 22.

YEAR ENDING DECEMBER 31, 1933

AMERICAN SOCIETY OF CIVIL ENGINEERS,
Disbursements for th fiscal year of the Society, ending December 31, 1933:
affairs of the Society.

Respectfully submitted,
GEORGE T. SEABURY,
Secretary.

DISBURSEMENTS

Salaries of Officers	$18 906.20
Retirement Allowances	1 048.84
Clerical Help	62 850.49
Traveling Allowance of Officers	11 794.26
Rent	10 191.96
Telephone	2 209.53
General Publications	5 580.89
General Printing	1 831.14
Postage	5 414.05
Binding	6 610.25
Badges	1 675.25
Certificates	120.94
Annual Prizes	402.47
Office Supplies	2 991.03
Furniture and Office Equipment	1 309.56
Current Business	3 322.41
Interest on Mortgage	1 200.00
Insurance	643.33
Reading Room	321.31
Miscellaneous	1 125.67
Employment Service	5 800.00
Library	7 194.00
American Standards Association	500.00
Local Sections	7 094.85
Technical Publications	101 518.07
Meetings	6 910.08
Technical Divisions	2 775.28
Technical Committees	7 146.51
Administrative Committees	591.71
Professional Committees	2 914.81
American Engineering Council	10 300.00
Construction League	1 632.88
Payment on Mortgage	12 000.00
The Freeman Fund:	
Principal (Purchase of Securities)	2 642.05
Income and Expense	1 306.40
The Alfred Noble Fund:	
Income and Expense	10.00
Loan to U. E. T.	5 000.00
E. W. L. (Held in escrow)	393.02
	$315 279.24
Cash on hand December 31, 1933	46 225.82*
	$361 505.06

Itemized Statement of Cash on Hand January 1, 1933

Society Funds in Chase National Bank 23d Street.	$14 248.97	
Society Funds in Chase National Bank 41st Street.	500.00	
Petty Cash (in hands of Secretary).............	5 000.00	$19 748.97
The 57th St. Property Fund.......................		11 923.82
Power Division		2 233.00
City Planning Division...........................		531.41
Surveying and Mapping Division...................		90.00
Special Committee on Stresses in Railroad Track............		60.28
Special Committee on Steel Column Research.................		333.70
Special Committee on Earths and Foundations................		2 351.59
In Escrow, E. W. L...............................		393.02
The Freeman Fund:		
Income and Expense........................	$39.87	
Principal	67.20	107.07
The Alfred Noble Fund:		
Uninvested	$189.39	
Not Released	55.00	
Income and Expense........................	822.74	1 067.13
The Merritt H. Smith Memorial Fund:		
Principal	$1 076.50	
Income and Expense........................	83.49	1 159 99
The Rudolph Hering Medal Fund:		
Principal	$455.97	
Income and Expense	59.36	515.33
		$40 515.31

Itemized Statement of Cash on Hand December 31, 1933

Society Funds in Chase National Bank 23d Street.	$11 580.74	
Society Funds in Chase National Bank 41st Street.	500 00	
Petty Cash (in hands of Secretary).............	5 000.00	$17 080.74
The 57th St. Property Fund.......................		22 966.32
Power Division		2 300.85
City Planning Division...........................		562.45
Surveying and Mapping Division...................		90.00
Special Committee on Stresses in Railroad Track............		60.28
Special Committee on Earths and Foundations................		1 141.00
The Freeman Fund:		
Principal		242.99
The Alfred Noble Fund:		
Not Released	....	55.00
The Merritt H. Smith Memorial Fund:		
Principal	$1 076.50	
Income and Expense........................	118.66	1 195.16
The Rudolph Hering Medal Fund:		
Principal	$455.97	
Income and Expense........................	75.06	531.03
		$46 225.82

APPLICATIONS FOR ADMISSION AND FOR TRANSFER

The Constitution provides that the Board of Direction shall elect or reject all applicants for *Admission* or for *Transfer,* and, in order to determine justly the eligibility of each candidate, the Board must depend largely upon the Membership for information.

This list is issued to members in every grade for the purpose of securing all such available information, and every member is urged to scan carefully each monthly list of candidates and to furnish the Board with data in regard to any applicant which may aid in determining his eligibility. It is the *Duty* of all *Members* to the *Profession* to assist the *Board* in this manner.

It is especially urged, in communications concerning applicants, that a *Definite Recommendation as to the Proper Grading in Each Case* be given, inasmuch as the grading must be based upon the opinions of those who know the applicant personally, as well as upon the nature and extent of his professional experience. If facts exist derogatory to the personal character or to the professional reputation of an applicant, they should be promptly communicated to the Board. *Communications Relating to Applicants are considered by the Board as Strictly Confidential.*

The Board of Direction will not consider the applications herein contained from residents of North America until the expiration of thirty (30) days, and from non-residents of North America until the expiration of ninety (90) days from **February 15, 1934.**

MINIMUM REQUIREMENTS FOR ADMISSION

Grade	General Requirement	Age	Length of Active Practice	Responsible charge of work
Member	Qualified to design as well as to direct important work	35 years	12 years*	5 years of important work
Associate Member	Qualified to direct work	27 years	8 years*	1 year
Junior	Qualified for sub-professional work	20 years†	4 years*	
Affiliate	Qualified by scientific acquirements or practical experience to co-operate with engineers	35 years	12 years*	5 years of important work
Fellow	Contributor to the permanent funds of the Society			

* Graduation from a school of engineering of recognized reputation is equivalent to 4 years of active practice.

† Membership ceases at age of 33 unless transferred to higher grade.

The fact that applicants give the names of certain members as references does not necessarily mean that such members endorse.

FOR ADMISSION

BERNEY, MARC (MIKE) PAUL, Sioux Falls, S. Dak. (Age 37.) Engr. in charge of Water-Pipe Dept., Tri-State Bldg. Materials Co. Refers to A. A. Chenoweth, C. H. Currie, C. A. McTaggart, E. H. Quinney, J. A. Thompson.

BREEDING, SETH DARNABY, Austin, Tex. (Age 33.) Asst. Engr., U. S. Geological Survey. Refers to C. S. Clark, A. C. Cook, A. H. Dunlap, C. E. Ellsworth, O. A. Faris, J. A. Norris.

BUCHHOLZ, JOHN ALBERT, Casper, Wyo. (Age 22.) Rodman, U. S. Bureau of Reclamation. Refers to R. D. Goodrich, E. K. Nelson, H. T. Person.

CALLAHAN, SAMUEL JOSEPH, Kansas City, Mo. (Age 41.) Constr. Engr. for County Court, Jackson County, Mo. Refers to E. W. Bacharach, D. M. Dodds, A. C. Everham, J. W. Ivy, J. C. Long.

CARLETON, ROBERT ANDREW WOOD, New York City. (Age 52.) Pres., Gen. Mgr. and Engr. in Chg., The Carleton Co., Inc., Engrs. and Contrs. Refers to F. N. Benedict, R. E. Dougherty, J. K. Finch, M. I. Killmer, G. Lindenthal, R. Ridgway, W. H. Yates.

CAROTHERS, HENRY PORTER, Austin, Tex. (Age 25.) Office Engr., Texas State Highway Dept. Refers to M. P. von Homeyer, G. R. Johnston, J. T. L. McNew, C. E. Sandstedt.

CAWLEY, CLIFFORD COMER, Los Angeles, Cal. (Age 24.) Senior Draftsman, Design Office, Los Angeles County Flood Control Dist. Refers to R. R. Martel, W. W. Michael, F. Thomas.

CHAMBERLIN, DONAL LEE, Cabin John, Md. (Age 36.) Vice-Pres. and Treasurer, Kennedy-Chamberlin Development Co., Archts., Engrs. and Bldrs. Refers to S. T. De La Mater, H. R. Hall, A. B. McDaniel, R. B. Morse, D. C. Walser.

CONOLE, CLEMENT VINCENT, Binghamton, N. Y. (Age 25.) Engr. and Foreman, U. S. Dept. of Interior. Refers to W. J. Farrisee, F. C. Wilson.

DETRICK, DANA FARRINGTON, Palo Alto, Cal. (Age 22.) Refers to E. L. Grant, C. Moser, L. B. Reynolds, E. C. Thomas.

DEWELL, ROBERT DIEVENDORF, Berkeley, Cal. (Age 23.) Draftsman, U. S. Coast & Geodetic Survey. Refers to A. W. Earl, B. A. Etcheverry, W. L. Huber, I. C. Steele, G. E. Troxell.

EDEN, EDWIN WINFIELD, JR., Highland Park, N. J. (Age 22.) Chf. of Survey Party, Middlesex County Mosquito Extermination Comm., Metuchen, N. J. Refers to A. Atkinson, H. N. Lendall.

FOGG, EDMUND KIMMEL, Panama City, Fla. (Age 46.) Div. Engr., Florida Road Dept., Tallahassee, Fla. Refers to R. L. Bannerman, A. Brest, J. D. Brown, C. B. Cooke, J. H. Dowling, G. B. Hills, H. J. Morrison, W. I. Nolen, J. R. Slade, C. E. Swank, L. E. Thornton, W. E. Wheat.

FRIEDMAN, JOSEPH LYON, Bronx, N. Y. (Age 21.) Refers to W. M. Fife, E. Mirabelli, P. W. Norton, C. M. Spofford.

FULTON, EDWARD ARTHUR, Clayton, Mo. (Age 35.) Cons. Engr.; member of firm, Kinsey Eng. Co., Pekin, Ill. and Clayton, Mo. Refers to F. Bachmann, T. R. Camp, J. N. Finlayson, K. Lindsey, G. R. Solomon, H. F. Wiedeman.

GARNELL, ALFRED WILLIAM, Spring Valley, N. Y. (Age 23.) Refers to J. B. Babcock, 3d, H. K. Barrows, C. B. Breed, W. M. Fife, C. M. Spofford.

GIBSON, ALEXANDER McKAY, West Philadelphia, Pa. (Age 29.) Senior Draftsman, Pennsylvania Dept. of Highways. Refers to J. E. Boatrite, T. Buckley, H. S. Hipwell, R. P. Ruger, C. H. Stevens, W. E. Witte.

GRAHAM, NATHAN JEROME, Oakland, Cal. (Age 31.) Refers to B. A. Etcheverry, C. G. Hyde.

HAHN, ARTHUR ROBERT, Nutley, N. J. (Age 27.) First Asst. to Borough Engr., Glen Ridge, N. J. Refers to H. N. Cummings, A. F. Eschenfelder, J. E. Garratt, R. E. Moss, J. H. Philips.

HARTH, WILFRID HENRY, Miami, Fla. (Age 24.) Jun. Officer, U. S. Coast and Geodetic Survey. Refers to H. Bouchard, L. C. Maugh.

HOLMES, JOSEPH MARK, Washington, D. C. (Age 29.) Jun. Engr., U. S. Geological Survey, Camdenton, Mo. Refers to G. D. Clyde, H. H. Hodgeson, O. W. Israelsen, J. G. Staack, R. B. West.

HORTON, JOHN, Sewickley, Pa. (Age 24.) With U. S. Geological Survey. Refers to B. L. Bigwood, N. C. Grover, A. H. Horton, J. C. Hoyt, A. N. Johnson, J. W. Mangan, C. G. Paulsen.

JUDGE, WILLIAM JOSEPH, Serres, Greece. (Age 56.) Constr. Mgr., John Monks & Sons-Ulen & Co. Refers to H. W. DeGraff, R. W. Gausmann, R. H. Keays, G. V. S. Keely, T. S. Shepperd, C. W. Sturtevant, H. A. Van Alstyne.

KAISER, EDGAR FOSBURGH, Washington, D. C. (Age 25.) Refers to F. T. Crowe, C. Derleth, Jr., E. Mead, J. L. Savage, B. W. Steele, R. F. Walter.

KINNEY, CHARLES WESLEY, Iowa City, Iowa. (Age 23.) Refers to W. C. McNown, H. A. Rice, D. L. Yarnell.

KISSAM, PHILIP, Princeton, N. J. (Age 37.) Asst. Prof. of Civ. Eng., Princeton Univ. Refers to J. L. Bauer, G. E. Beggs, F. H. Constant, J. J. Vail, S. F. Voorhees, F. N. Willson.

KNAPP, RONALD CLARENCE, Olympia, Wash. (Age 43.) Bridge Designer, Bridge Div., Washington State Highway Dept. Refers to C. E. Cleaver, F. C. Dunham, O. R. Elwell, M. A. Gould, L. V. Murrow, R. K. Tiffany, M. S. Woodin.

KNOLL, CARL ALVIN, Los Angeles, Cal. (Age 25.) Jun. Engr., Metropolitan Water Dist. of Southern California. Refers to H. R. Bolton, H. Jones, R. E. Rule, W. E. Whittier.

KRAMER, ROBERT WILLIAM, Columbus, Ohio. (Age 28.) Refers to C. T. Morris, J. R. Shank.

LEE, HARRY HARRISON, JR., Media, Pa. (Age 26.) Jun. Constr. Inspector, Pennsylvania Dept. of Highways. Refers to H. B. Shattuck, E. D. Walker.

LOWY, FREDERICK CHARLES, New York City. (Age 34.) Refers to W. J. Ash, C. L. Crandall, R. Smillie, R. H. Vanderbrook, E. Welle.

ORTENBLAD, ALBERTO, Rio de Janeiro, Brazil. (Age 33.) Pres., Companhia Brasileira Melhoramentos e Construccoes (Brazilian Impvt. & Constr. Co.). Refers to G. M. de Menezes, R. C. Simonsen.

OWENS, JOHN CUSTER, Chevy Chase, Md. (Age 29.) Refers to G. P. Boomsliter, L. V. Carpenter, R. P. Davis, W. S. Downs, T. S. Lang, M. W. Smith, Jr.

PEARSON, EDWARD RUSSELL, Vicksburg, Miss. (Age 26.) Jun. Engr., U. S. Engr. Office, 2d New Orleans Dist. Refers to J. A. C. Callan, F. C. Carey, O. N. Floyd, H. D. Moore, J. D. Walker.

PEEL, HARRY HERBERT, Paris, Tex. (Age 35.) Office Engr., Texas State Highway Dept. Refers to E. P. Arneson, F. M. Davis, W. A. King, J. G. Lott, E. J. McCaustland, J. E. Pirie.

PETERSEN, LAWRENCE CHRISTIAN, Trenton, N. J. (Age 42.) Designer of Bridges and Structures, New Jersey State Highway Comm., Bridge Div. Refers to C. S. Bissell, F. C. Claus, M. Goodkind, S. Johannesson, C. A. Mead, S. A. Snook, J. L. Vogel.

PRUDEN, WORRELL FRANZONI, Azusa, Cal. (Age 23.) Chainman, Los Angeles County Flood Control Dist. Refers to S. M. Fisher, K. J. Harrison, R. R. Martel, W. W. Michael, F. Thomas.

ROBERTS, DWIGHT FULTON, Fullerton, Cal. (Age 32.) Refers to T. R. Agg, G. F. Burch, H. F. Clemmer, O. L. Gearhart, A. Marston, H. W. Russell, A. E. Stoddard.

RUDDER, IRVING AVRUM, Brooklyn, N. Y. (Age 26.) Refers to E. G. Hooper, T. Saville.

SUTPHIN, FRANCIS STOWERS, San Marino, Cal. (Age 44.) Senior Design Draftsman, Los Angeles County Flood Control Dist. Refers to J. C. Fitterer, N. B. Hodgkinson, A. S. Kemman, F. C. McMillan, W. W. Patch, H. S. Rogers, J. V. V. Spielman, I. S. Voorhees, J. E. Waite.

TANDY, MILAM FLACK, Glen Ferris, W. Va. (Age 29.) Asst. to Chf. Engr., New Kanawha Power Co. Refers to H. K. Barrows, L. H. Davis, J. W. Hall, O. M. Jones, G. E. Russell, J. E. Settle, C. M. Spofford.

TINGEY, WILLIS ALMA, St. James, Mo. (Age 29.) Jun. Engr. (Topographic), U. S. Geological Survey, Washington, D. C. Refers to O. W. Israelsen, C. L. Sadler, J. G. Staack, R. B. West, L. M. Winsor.

TUTEWILER, HARVEY ALFORD, Indianapolis, Ind. (Age 43.) Pres., Gen. Mgr. and Owner of Ready Mixed Concrete Corporation and Standard Paving Co. Refers to T. H. David, H. O. Garman, C. M. Geupel, J. E. Hall, C. H. Hurd.

VANCE, JAMES DAVIS, Harrisonburg, Va. (Age 24.) Chf. of Party, U. S. Coast and Geodetic Survey. Refers to J. A. Anderson, M. J. Bussard.

VAN ORMAN, CLARE RALSTON, Kansas City, Kans. (Age 30.) Jun. Engr., U. S. Engr. Office, Missouri River Div., Hydr. Sec. Refers to F. W. Epps, W. C. McNown, H. A. Marshall, H. A. Rice, L. B. Smith.

FOR TRANSFER

FROM THE GRADE OF ASSOCIATE MEMBER

BARNES, ROBERT LELAND, Assoc. M., Chalmette, La. (Elected April 27, 1931.) (Age 37.) Res. Engr., Louisiana Highway Comm. Refers to R. P. Boyd, J. A. Bruce, H. B. Henderlite, N. E. Lant, J. M. Page, N. C. Woody, K. R. Young, W. C. Youngs.

BUTLER, MERRILL, Assoc. M., Los Angeles, Cal. (Elected July 9, 1923.) (Age 42.) Deputy City Engr., Bureau of Eng., City of Los Angeles. Refers to H. P. Cortelyou, M. C. Halsey, P. B. Harris, T. Maddock, J. O. Marsh, R. J. Reed, R. W. Stewart, H. A. Van Norman.

GILL, GRAYSON WOODWARD, Assoc. M., Dallas, Tex. (Elected July 11, 1927.) (Age 40.) Archt. Refers to S. P. Finch, R. O. Jameson, J. H. Knox, O. H. Koch, W. H. Meier, E. N. Noyes, W. J. Powell, G. V. Rhines, J. J. Richey.

LEWIS, DUDLEY LELAND, Assoc. M., Fort Worth, Tex. (Elected May 28, 1923.) (Age 48.) City Engr. Refers to J. H. Brillhart, C. M. Davis, H. B. Friedman, P. M. Geren, S. A. Greeley, J. H. Gregory, J. B. Hawley.

MURRAY, HOWARD SLATER, Assoc. M., Las Vegas, Nev. (Elected Junior Nov. 12, 1928; Assoc. M. June 9, 1930.) (Age 35.) Desiginng Engr. with Six Companies, Inc., Contrs. on Boulder Dam. Refers to A. H. Ayers, R. A. Beebee, F. T. Crowe, R. E. Davis, C. Derleth, Jr., H. D. Dewell, G. B. Hegardt.

UNGER, GEORGE FREDERICK, Assoc. M., Niagara Falls, N. Y. (Elected Jan. 17, 1921.) (Age 47.) Executive Engr., Niagara Reservation Comm. Refers to A. J. Dillenbeck, W. T. Huber, E. P. Lupfer, J. T. Mockler, H. E. Riexinger, F. K. Wing.

FROM THE GRADE OF JUNIOR

BEATTY, ROBERT, Jun., Philadelphia, Pa. (Elected May 19, 1924.) (Age 32.) Field Draftsman, Bureau of Eng. & Surveys, Div. of Zoning, City of Philadelphia. Refers to H. C. Berry, E. W. Denzler, Jr., M. Duncan, A. F. Holler, H. G. Leng, J. B. Myers, R. J. Wolf.

HARRJE, HENRY JOHN, Jun., Buffalo, N. Y. (Elected Nov. 23, 1931.) (Age 32.) Chf. Engr., Federal Eng. & Constr. Co. Refers to R. L. Barbehenn, A. R. C. Markl, M. I. Merritt, T. M. Ripley, H. W. Weitzner, C. A. Young.

KOENITZER, LESTER HENRY, Jun., Manhattan, Kans. (Elected July 11, 1927.) (Age 32.) Instructor, Applied Mechanics Dept., Kansas State Coll. Refers to T. R. Agg, L. E. Conrad, A. H. Fuller, M. W. Furr, F. Kerekes, C. H. Scholer.

McATEE, FRAYNE LEIGH, Jun., Boise, Idaho. (Elected Oct. 26, 1931.) (Age 32.) Inspector, Chf. of Party, Bureau of Highways, Dept. of Public Works, State of Idaho. Refers to A. C. Blomgren, I. C. Crawford, J. E. Hayes, R. B. Ketchum, F. H. Pickett, R. W. Wilson.

PETERSON, EDWARD JOHN LAWRENCE, Jun., Marysville, Cal. (Elected Oct. 1, 1926.) (Age 32.) Asst. Maintenance Engr., Div. of Highways, Dist. III, Sacramento, Cal. Refers to J. C. L. Fish, F. J. Grumm, C. Moser, C. S. Pope, L. B. Reynolds, T. E. Stanton, Jr., C. B. Wing.

RICE, PAUL PRESTON, Jun., Easton, Pa. (Elected Oct. 14, 1929.) (Age 31.) Instructor in Civ. Eng., Lafayette Coll. Refers to W. Bowie, A. S. Cutler, C. T. Johnston, W. S. Lohr, H. W. Mixsell, R. S. Patton, L. Perry, E. H. Rockwell, F. W. Slantz, C. E. Tilton

SALMOND, IRVING MUNRO, Jun., Ann Arbor, Mich. (Elected March 11, 1929.) (Age 32.) Bridge Draftsman, Bridge Office, Michigan Central R. R. Refers to J. E. Bebb, O. H. S. Koch, C. A. Melick, T. J. Mitchell, R. L. Morrison, M. F. Ohr.

SMITH, FRANK MILLER, JR., Jun., Arlington, Tex. (Elected Nov. 14, 1927.) (Age 32.) Associate Prof. of Civ. Eng., North Texas Agricultural Coll. Refers to T. C. Forrest, Jr., J. D. Fowler, J. H. Knox, O. H. Koch, E. L. Myers, E. N. Noyes, J. J. Richey.

TWICHELL, TRIGG, Jun., Austin, Tex. (Elected June 7, 1926.) (Age 32.) Associate Engr., Water Resources Branch, U. S. Geological Survey. Refers to C. S. Clark, A. H. Dunlap, C. E. Ellsworth, N. C. Grover, C. E. McCashin, J. A. Norris, O. A. Seward, Jr.

VERNON, ABNER JOSEPH, Jun., Jersey City, N. J. (Elected Nov. 14, 1927.) (Age 32.) Refers to E. H. Aldrich, J. G. Basinger, P. C. Gillette, N. I. Kass, A. T. Ricketts.

WILSON, WILLIAM SIDNEY, Jun., Banning, Cal. (Elected Oct. 14, 1930.) (Age 32.) Jun. Engr., Metropolitan Water Dist. of Southern California. Refers to N. Aanonsen, C. B. Andrews, M. H. Benagh, J. B. Bond, F. W. Hough, H. W. Jorgensen, A. R. Keller.

The Board of Direction will consider the applications in this list not less than thirty days after the date of issue.